Formation and Collapse of Nonaxisymmetric Protostellar Cores in Magnetic Interstellar Clouds

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Magnetic Support of Clouds

Evidence for magnetic support of molecular clouds:
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- Polarimetry.
Magnetic Support of Clouds

Hildebrand et al. (2000)
Magnetic Support of Clouds

NGC 1333 IRAS 4A (Girart, Rao, & Marrone 2006)
Magnetic Support of Clouds

Further evidence:

- Modified Chandrasekhar-Fermi method.
- Zeeman splitting.

The measured magnetic fields yield mass-to-ux ratios $M_B$ (s_B) that are within a factor 2 below or above the critical value for gravitational collapse (Crutcher 2004).
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The measured magnetic fields yield mass-to-flux ratios $M/\Phi_B \ (= \sigma_n/B)$ that are within a factor $\sim 2$ below or above the critical value for gravitational collapse (Crutcher 2004).
Past decade: axisymmetric models of formation and collapse of protostellar cores in magnetically supported molecular clouds were developed.

Ciolek & Basu (2000) applied a model to the L1544 prestellar core.
- Reproduced observed density and velocity profiles (Williams et al. 1999, Caselli et al. 2002)
- Predicted magnetic field strength, confirmed by Zeeman measurements (Crutcher & Troland 2000).

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L1544 Prestellar Core (Williams et al. 1999)
Dimensionless initial mass-to-flux ratio for a self-gravitating object:

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\mu_0 \equiv \frac{(M/\Phi_B)_0}{(M/\Phi_B)_{\text{crit}}},
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where \((M/\Phi_B)_{\text{crit}} \simeq 0.17/\sqrt{G}\) is the critical mass-to-flux ratio.
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\(\mu_0 < 1 \Rightarrow \text{Subcritical, magnetically supported.}\)

\(\mu_0 > 1 \Rightarrow \text{Supercritical, gravitational collapse.}\)
Dimensionless neutral-ion collision time:

\[
\tilde{\tau}_{ni,0} \equiv \frac{\text{neutral} - \text{ion collision time}}{\text{gravitational contraction timescale}}
\]

\[
= 0.26 \left( \frac{10^3 \text{ cm}^{-3}}{n_n} \right)^{1/2} \left( \frac{10^{-7}}{x_i} \right),
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where \( x_i = n_i/n_n \) is the degree of ionization.
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For typical cloud conditions, \( \tilde{\tau}_{ni,0} \approx 0.2 \).
Modeling Nonaxisymmetric Collapse

Model Cloud Schematic

\[ Z(x,y) \]

\[ P_{\text{ext}} \]

\( B \)
Modeling Nonaxisymmetric Collapse

We numerically integrate the coupled nonlinear partial differential equations for the system of neutral and ion fluids, and the magnetic field in Cartesian geometry. These include:

- Continuity of mass.
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- Maxwell’s equations.
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Assume small-amplitude perturbation $\delta f$ for any physical variable $f$ in the system of equations governing the evolution of a model cloud:

$$\delta f(x, y, t) \propto \exp(i[k_x x + k_y y - \omega t]) .$$
Linearized and Fourier-Analyzed System

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Gravitationally unstable modes exist if $\Re[\omega]$ is positive. The growth time for the instability is

$$
\tau_g = \frac{1}{\Re[\omega]}.
$$
Timescale for Gravitational Instability

\[ \tilde{\tau}_{\text{ni,0}} = 0.2 \]

\[ \mu_0 = 0.5, 0.8, 1, 1.1, 2 \]

\[ \tau_g \]

\[ \lambda \]
Lengthscale of Maximum Growth Rate $\lambda_{g,m}$
Numerical Simulations of Nonaxisymmetric Core Formation
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- Nonlinear simulations verify that the fundamental fragmentation scale is \( \lambda_{g,m} \).
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- From a linear analysis of the models, determined the lengthscale of maximum gravitational instability $\lambda_{g,m}$ and its dependence on the parameter $\mu_0$. $\lambda_{g,m}$ has a resonance for clouds near the critical state $\mu_0 \sim 1$.
- Nonlinear simulations verify that the fundamental fragmentation scale is $\lambda_{g,m}$.
- Clouds with $\mu_0 \gtrsim 2$ have cores with large-scale infall velocities that are excluded by observations.