FORCE VECTORS, VECTOR OPERATIONS & ADDITION COPLANAR FORCES

Today’s Objective:

Students will be able to:

a) Add 2-D vectors using Cartesian vector notations.

b) Represent a 3-D vector in a Cartesian coordinate system.

c) Find the magnitude and coordinate angles of a 3-D vector

In-Class activities:

- Application of Adding Forces
- Parallelogram Law
- Resolution of a Vector Using Cartesian Vector Notation (CVN)
- Addition Using CVN
CARTESIAN VECTORS AND
THEIR ADDITION & SUBTRACTION

In-Class Activities:

- Applications / Relevance
- A Unit Vector
- 3-D Vector Terms
- Adding Vectors
- Concept Quiz
- Examples
APPLICATION OF VECTOR ADDITION

There are three concurrent forces acting on the hook due to the chains.

We need to decide if the hook will fail (bend or break).

To do this, we need to know the resultant or total force acting on the hook as a result of the three chains.
VECTOR ADDITION USING EITHER THE PARALLELOGRAM LAW OR TRIANGLE

Parallelogram Law:

Triangle method (always ‘tip to tail’):

How do you subtract a vector?

How can you add more than two concurrent vectors graphically?
“Resolution” of a vector is breaking up a vector into components.

It is kind of like using the parallelogram law in reverse.
ADDITION OF A SYSTEM OF COPLANAR FORCES
(Section 2.4)

- We ‘resolve’ vectors into components using the x and y-axis coordinate system.

- Each component of the vector is shown as a magnitude and a direction.

- The directions are based on the x and y axes. We use the “unit vectors” $\mathbf{i}$ and $\mathbf{j}$ to designate the x and y-axes.
For example,

\[ \mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} \quad \text{or} \quad \mathbf{F}' = F'_x \mathbf{i} + (-F'_y) \mathbf{j} \]

The x and y axis are always perpendicular to each other. Together, they can be directed at any inclination.
• Step 1 is to resolve each force into its components.

• Step 2 is to add all the x-components together, followed by adding all the y-components together. These two totals are the x and y-components of the resultant vector.

• Step 3 is to find the magnitude and angle of the resultant vector.
An example of the process:

\[ \mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} \]

\[ = F_{1x} \mathbf{i} + F_{1y} \mathbf{j} - F_{2x} \mathbf{i} + F_{2y} \mathbf{j} + F_{3x} \mathbf{i} - F_{3y} \mathbf{j} \]

\[ = (F_{1x} - F_{2x} + F_{3x}) \mathbf{i} + (F_{1y} + F_{2y} - F_{3y}) \mathbf{j} \]

\[ = (\mathbf{F_{Rx}}) \mathbf{i} + (\mathbf{F_{Ry}}) \mathbf{j} \]
You can also represent a 2-D vector with a magnitude and angle.

\[ \theta = \tan^{-1}\left| \frac{F_{Ry}}{F_{Rx}} \right| \quad \text{and} \quad \mathbf{F}_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \]
**EXAMPLE**

**Given:** Three concurrent forces acting on a tent post.

**Find:** The magnitude and angle of the resultant force.

**Plan:**

a) **Resolve** the forces into their x-y components.

b) **Add** the respective **components** to get the resultant vector.

c) Find **magnitude** and **angle** from the resultant components.
\( \mathbf{F}_1 = \{0 \mathbf{i} + 300 \mathbf{j}\} \text{ N} \)

\( \mathbf{F}_2 = \{-450 \cos (45^\circ) \mathbf{i} + 450 \sin (45^\circ) \mathbf{j}\} \text{ N} \)

\[= \{-318.2 \mathbf{i} + 318.2 \mathbf{j}\} \text{ N} \]

\( \mathbf{F}_3 = \{(3/5) \times 600 \mathbf{i} + (4/5) \times 600 \mathbf{j}\} \text{ N} \)

\[= \{360 \mathbf{i} + 480 \mathbf{j}\} \text{ N} \]
Summing up all the \( i \) and \( j \) components respectively, we get,

\[
\mathbf{F_R} = \{ (0 - 318.2 + 360) \mathbf{i} + (300 + 318.2 + 480) \mathbf{j} \} \text{ N}
\]

\[
= \{ 41.80 \mathbf{i} + 1098 \mathbf{j} \} \text{ N}
\]

Using magnitude and direction:

\[
\mathbf{F_R} = \sqrt{(41.80^2 + (1098)^2)} = 1099 \text{ N}
\]

\[
\phi = \tan^{-1}(1098/41.80) = 87.8^\circ
\]
GROUP  PROBLEM SOLVING

**Plan:**

a) **Resolve** the forces into their x and y-components.

b) **Add** the respective **components** to get the resultant vector.

c) Find **magnitude** and **angle** from the resultant components.

**Given:** Three concurrent forces acting on a bracket.

**Find:** The magnitude and angle of the resultant force.
\[ F_1 = \{800 \cos (60°) \, i + 800 \sin (60°) \, j \} \, N \]

\[ = \{ 400 \, i + 692.8 \, j \} \, N \]

\[ F_2 = \{-600 \sin (45°) \, i + 600 \cos (45°) \, j \} \, N \]

\[ = \{-424.3 \, i + 424.3 \, j \} \, N \]

\[ F_3 = \{(12/13) \, 650 \, i - (5/13) \, 650 \, j \} \, N \]

\[ = \{ 600 \, i - 250 \, j \} \, N \]
Summing up all the $i$ and $j$ components, respectively, we get,

$$\mathbf{FR} = \{ (400 - 424.3 + 600) \mathbf{i} + (692.8 + 424.3 - 250) \mathbf{j} \} \text{N}$$

$$= \{ 575.7 \mathbf{i} + 867.1 \mathbf{j} \} \text{N}$$

Now find the magnitude and angle,

$$\mathbf{FR} = \left( \left( \frac{2}{575.7} \right)^2 + \left( \frac{2}{867.1} \right)^2 \right)^{1/2} = 1041 \text{ N}$$

$$\phi = \tan^{-1} \left( \frac{867.1}{575.7} \right) = 56.4^\circ$$

From positive x-axis, $\phi = 56.4^\circ$
Many structures and machines involve 3-dimensional space.

In this case, the power pole has guy wires helping to keep it upright in high winds. How would you represent the forces in the cables using Cartesian vector form?
In the case of this radio tower, if you know the forces in the three cables, how would you determine the resultant force acting at D, the top of the tower?
CARTESIAN UNIT VECTORS

For a vector $A$, with a magnitude of $A$, an unit vector is defined as

$$uA = A / A.$$ 

Characteristics of a unit vector:

a) Its magnitude is 1.

b) It is dimensionless (has no units).

c) It points in the same direction as the original vector ($A$).

The unit vectors in the Cartesian axis system are $i$, $j$, and $k$. They are unit vectors along the positive $x$, $y$, and $z$ axes respectively.
CARTESIAN VECTOR REPRESENTATION

Consider a box with sides AX, AY, and AZ meters long.

The vector \( \mathbf{A} \) can be defined as

\[
\mathbf{A} = (AX \mathbf{i} + AY \mathbf{j} + AZ \mathbf{k}) \text{ m}
\]

The projection of vector \( \mathbf{A} \) in the x-y plane is \( \mathbf{A}' \). The magnitude of \( \mathbf{A}' \) is found by using the same approach as a 2-D vector:

\[
\mathbf{A}' = (AX^2 + AY^2)^{1/2}
\]

The magnitude of the position vector \( \mathbf{A} \) can now be obtained as

\[
\mathbf{A} = ((\mathbf{A}')^2 + AZ^2)^{1/2} = (AX^2 + AY^2 + AZ^2)^{1/2}
\]
Using trigonometry, “direction cosines” are found using

\[ \cos \alpha = \frac{A_x}{A} \quad \cos \beta = \frac{A_y}{A} \quad \cos \gamma = \frac{A_z}{A} \]

These angles are not independent. They must satisfy the following equation.

\[ \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \]

This result can be derived from the definition of a coordinate direction angles and the unit vector. Recall, the formula for finding the unit vector of any position vector:

\[ \mathbf{u}_A = \frac{A}{|A|} = \frac{A_x}{A} \mathbf{i} + \frac{A_y}{A} \mathbf{j} + \frac{A_z}{A} \mathbf{k} \]

or written another way, \( \mathbf{u}_A = \cos \alpha \mathbf{i} + \cos \beta \mathbf{j} + \cos \gamma \mathbf{k} \).
ADDITION OF CARTESIAN VECTORS

(Section 2.6)

Once individual vectors are written in Cartesian form, it is easy to add or subtract them. The process is essentially the same as when 2-D vectors are added.

\[
\mathbf{F}_R = \sum \mathbf{F} = \sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k}
\]

For example, if

\[
\mathbf{A} = AX \mathbf{i} + AY \mathbf{j} + AZ \mathbf{k} \quad \text{and}
\]

\[
\mathbf{B} = BX \mathbf{i} + BY \mathbf{j} + BZ \mathbf{k}, \quad \text{then}
\]

\[
\mathbf{A} + \mathbf{B} = (AX + BX) \mathbf{i} + (AY + BY) \mathbf{j} + (AZ + BZ) \mathbf{k}
\]

or

\[
\mathbf{A} - \mathbf{B} = (AX - BX) \mathbf{i} + (AY - BY) \mathbf{j} + (AZ - BZ) \mathbf{k}.
\]
IMPORTANT NOTES

Sometimes 3-D vector information is given as:

a) Magnitude and the coordinate direction angles, or,

b) Magnitude and projection angles.

You should be able to use both these sets of information to change the representation of the vector into the Cartesian form, i.e.,

\[ \mathbf{F} = \{10 \ i - 20 \ j + 30 \ k\} \ \text{N} \ . \]
EXAMPLE

Given: Two forces $F_1$ and $F_2$ are applied to a hook.

Find: The resultant force in Cartesian vector form.

Plan:

1) Using geometry and trigonometry, write $F_1$ and $F_2$ in Cartesian vector form.

2) Then add the two forces (by adding x and y-components).
Solution:

First, resolve force $F_1$.

\[ F_x = 0 = 0 \text{ lb} \]

\[ F_y = 500 \left( \frac{4}{5} \right) = 400 \text{ lb} \]

\[ F_z = 500 \left( \frac{3}{5} \right) = 300 \text{ lb} \]

Now, write $F_1$ in Cartesian vector form (don’t forget the units!).

\[ F_1 = \{0 \, i + 400 \, j + 300 \, k\} \text{ lb} \]
Now, resolve force $F_2$.

$F_{2z} = -800 \sin 45^\circ = -565.7 \text{ lb}$

$F_{2'} = 800 \cos 45^\circ = 565.7 \text{ lb}$

$F_{2'}$ can be further resolved as,

$F_{2x} = 565.7 \cos 30^\circ = 489.9 \text{ lb}$

$F_{2y} = 565.7 \sin 30^\circ = 282.8 \text{ lb}$

Thus, we can write:

$F_2 = \{489.9 \mathbf{i} + 282.8 \mathbf{j} - 565.7 \mathbf{k}\} \text{ lb}$
So \( \mathbf{FR} = \mathbf{F1} + \mathbf{F2} \) and

\[
\mathbf{F1} = \{0 \, \mathbf{i} + 400 \, \mathbf{j} + 300 \, \mathbf{k}\} \text{ lb}
\]

\[
\mathbf{F2} = \{489.9 \, \mathbf{i} + 282.8 \, \mathbf{j} - 565.7 \, \mathbf{k}\} \text{ lb}
\]

\[
\mathbf{FR} = \{490 \, \mathbf{i} + 683 \, \mathbf{j} - 266 \, \mathbf{k}\} \text{ lb}
\]
GROUP PROBLEM SOLVING

Given: The screw eye is subjected to two forces, $F_1$ and $F_2$.

Find: The magnitude and the coordinate direction angles of the resultant force.

Plan:

1) Using the geometry and trigonometry, resolve and write $F_1$ and $F_2$ in the Cartesian vector form.

2) Add $F_1$ and $F_2$ to get $FR$.

3) Determine the magnitude and angles $\alpha$, $\beta$, $\gamma$.
GROUP PROBLEM SOLVING (continued)

First resolve the force $F_1$.

$$F_{1z} = -250 \sin 35^\circ = -143.4 \text{ N}$$

$$F' = 250 \cos 35^\circ = 204.8 \text{ N}$$

$F'$ can be further resolved as,

$$F_{1x} = 204.8 \sin 25^\circ = 86.6 \text{ N}$$

$$F_{1y} = 204.8 \cos 25^\circ = 185.6 \text{ N}$$

Now we can write:

$$F_1 = \{86.6 \hat{i} + 185.6 \hat{j} - 143.4 \hat{k}\} \text{ N}$$
The force $\vec{F}_2$ can be represented in the Cartesian vector form as:

$$\vec{F}_2 = 400 \{ \cos 120^\circ \, \vec{i} + \cos 45^\circ \, \vec{j} + \cos 60^\circ \, \vec{k} \} \, \text{N}$$

$$\quad = \{ -200 \, \vec{i} + 282.8 \, \vec{j} + 200 \, \vec{k} \} \, \text{N}$$

$$\vec{F}_2 = \{ -200 \, \vec{i} + 282.8 \, \vec{j} + 200 \, \vec{k} \} \, \text{N}$$

Now, resolve force $\vec{F}_2$. 
GROUP PROBLEM SOLVING (continued)

Now find the magnitude and direction angles for the vector.

So \( \mathbf{FR} = \mathbf{F1} + \mathbf{F2} \) and

\[
\mathbf{F1} = \{ 86.6 \mathbf{i} + 185.6 \mathbf{j} - 143.4 \mathbf{k} \} \text{ N}
\]

\[
\mathbf{F2} = \{ -200 \mathbf{i} + 282.8 \mathbf{j} + 200 \mathbf{k} \} \text{ N}
\]

\[
\mathbf{FR} = \{ -113.4 \mathbf{i} + 468.4 \mathbf{j} + 56.6 \mathbf{k} \} \text{ N}
\]

\[
\mathbf{FR} = \{ (-113.4)^2 + 468.4^2 + 56.6^2 \}^{1/2} = 485.2 = 485 \text{ N}
\]

\[
\alpha = \cos^{-1} \left( \frac{\mathbf{FR}_x}{\mathbf{FR}} \right) = \cos^{-1} \left( \frac{-113.4}{485.2} \right) = 103.5^\circ
\]

\[
\beta = \cos^{-1} \left( \frac{\mathbf{FR}_y}{\mathbf{FR}} \right) = \cos^{-1} \left( \frac{468.4}{485.2} \right) = 15.1^\circ
\]

\[
\gamma = \cos^{-1} \left( \frac{\mathbf{FR}_z}{\mathbf{FR}} \right) = \cos^{-1} \left( \frac{56.6}{485.2} \right) = 83.3^\circ
\]