**N.B.:** You will be graded on 5 problems, 20 points per problem. Problems 1, 2, and 3 are mandatory and will be graded. Before turning in your exam, please make sure you have circled the two problems you want to be graded out of problems 4, 5 and 6.
Problem #1 (20)

A frame supports a 300 lb load as shown

(a) Draw a free body diagram of the entire frame; (3pts)
(b) Determine the support reactions at A and C; (6pts)
(c) Draw free body diagrams of members ABC and CEF; (4pts)
(d) Determine the horizontal and vertical components of force at C which member ABC exerts on member CEF. (7pts)

Solution:

(b) \[ \sum M_A = F_y \cdot 6 - T \cdot 10 = 0 \Rightarrow F_y = 500\text{lb} \] (2 pts)
\[ \sum F_x = A_x = 0 \] (2 pts)
\[ \sum F_y = F_y + A_y - T = 0 \Rightarrow A_y = -200\text{lb} \] (2 pts)

(c) FBD of member CEF (2 pts):

(d) \[ \sum M_B = C_y \cdot 4 - T \cdot 1 = 0 \Rightarrow C_x = -75\text{lb} \] (3 pts)
\[ \sum M_B = C_x \cdot 4 - C_y \cdot 3 - A_y \cdot 3 = 0 \Rightarrow C_y = 100\text{lb} \] (4 pts)
Problem 2 (20 points)

Part 1: Consider the following linear system of equations.

\[3p - 7q + 4r = 3\]
\[2p + 5q - 5r = -2\]
\[7p - 9q + 3r = 7\]

(a) Write the system of equations in matrix form, \(AX = B\), and identify \(A\), \(X\), and \(B\).

(b) Determine the classical adjoint of the matrix \(A\); that is, compute \(\text{adj}(A)\).

(c) Find the inverse of \(A\) using the adjoint-matrix method.

Part 2: Use Cramer’s rule to solve for the unknown \(z\) in the following linear system; note that \(\alpha\) is an arbitrary parameter.

\[
\begin{bmatrix}
9 & 2 & \alpha \\
1 & 0 & 2 \\
3 & 0 & 4 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix}
= 
\begin{bmatrix}
8 \\
5 \\
-3 \\
\end{bmatrix}
\]

Note: You must show all work to receive full credit.

\[\begin{array}{l}
\text{(a)} \quad \begin{bmatrix}
3 & -7 & 4 \\
2 & 5 & -5 \\
7 & -9 & 3 \\
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r \\
\end{bmatrix}
= 
\begin{bmatrix}
3 \\
-2 \\
7 \\
\end{bmatrix} \\
\text{As shown,}
\end{array}
\]

(b) The minors of \(A\) are

\[
\begin{align*}
M_{11} &= \begin{vmatrix}
2 & -5 \\
-3 & 7 \\
\end{vmatrix} = -30, & M_{12} &= \begin{vmatrix}
3 & 7 \\
-3 & 7 \\
\end{vmatrix} = 41, & M_{13} &= \begin{vmatrix}
3 & -7 \\
-3 & 7 \\
\end{vmatrix} = -53 \\
M_{21} &= \begin{vmatrix}
-7 & 4 \\
-3 & 7 \\
\end{vmatrix} = 15, & M_{22} &= \begin{vmatrix}
3 & 7 \\
3 & 7 \\
\end{vmatrix} = -19, & M_{23} &= \begin{vmatrix}
3 & -7 \\
3 & 7 \\
\end{vmatrix} = 22 \\
M_{31} &= \begin{vmatrix}
-7 & 4 \\
5 & -5 \\
\end{vmatrix} = 15, & M_{32} &= \begin{vmatrix}
3 & 7 \\
5 & -5 \\
\end{vmatrix} = -23, & M_{33} &= \begin{vmatrix}
3 & -7 \\
5 & -5 \\
\end{vmatrix} = 29 \\
\end{align*}
\]

Thus, the cofactor matrix is

\[
C = \begin{bmatrix}
-30 & -41 & -53 \\
-15 & -19 & -22 \\
15 & 23 & 27 \\
\end{bmatrix}
\Rightarrow
\text{adj}(A) = \begin{bmatrix}
-30 & -15 & 15 \\
-41 & -19 & 23 \\
-53 & -22 & 27 \\
\end{bmatrix}
\]

(c) \(\text{inv}(A) = -\frac{1}{\det(A)} \text{adj}(A)\)

\[\begin{align*}
\text{inv}(A) &= -\frac{1}{15} \begin{bmatrix}
-30 & -15 & 15 \\
-41 & -19 & 23 \\
-53 & -22 & 27 \\
\end{bmatrix} \\
&= \begin{bmatrix}
2 & 1 & -1 \\
4/15 & 1/15 & -2/15 \\
53/15 & 22/15 & -27/15 \\
\end{bmatrix}
\end{align*}
\]

Part 2:

\[
z = \frac{\det(A_3)}{\det(A)} = 9
\]

where

\[
\det(A_3) = \begin{vmatrix}
9 & 2 & 8 \\
1 & 0 & 5 \\
3 & 0 & -3 \\
\end{vmatrix} = 36,
\det(A) = \begin{vmatrix}
9 & 2 & \alpha \\
1 & 0 & 2 \\
3 & 0 & 4 \\
\end{vmatrix} = 4
\]

\[
A^{-1} \approx \begin{bmatrix}
2.0 & 1.0 & -1.0 \\
2.73 & 1.267 & -1.53 \\
3.53 & 1.467 & -1.93 \\
\end{bmatrix}
\]

\[
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2.0 & 1.0 & -1.0 \\
2.73 & 1.267 & -1.53 \\
3.53 & 1.467 & -1.93 \\
\end{bmatrix}
\]
Problem 3 (20 points)

Note: You need to draw all required FBD’s for this problem

Problem 3 (20 points)

Three blocks A, C and E are connected by two ropes R₁ and R₂, as shown in the figure below. The first rope passes under a smooth peg at B and connects blocks A and C. The second rope passes over a peg at D and connects blocks C and E. The friction coefficient between the peg at D and the rope is 0.3. Block A has a weight of 600 N and rests on a horizontal surface. Block C has a weight of 200 N and rests on a surface making an angle of 30° with the horizontal. The coefficients of friction between blocks A and C and the supporting surfaces are equal to 0.25. Block E has a weight Wₑ that is leading to an impending downward motion.

1. What is the tension in rope R₁ when block A is in a condition of impending sliding?
2. Verify that block A will not tip.
3. Will block C tip or slip (justify fully your answer)?
4. What is the tension in rope R₂ between block C and peg D?
5. What is the weight Wₑ of block E?

What is the tension in rope R₁ when block A

FBD of A

\[ \Sigma F_y = 0 \]
\[ Na - Wa = Na - 600 = 0 \quad \rightarrow \quad Na = 600 \text{ N} \]

\[ Fa = \mu Na = 0.25 \times 600 = 150 \text{ N} \]

\[ \Sigma F_x = 0 \]
\[ Tac - Fa = Ta - 150 = 0 \quad \rightarrow \quad \text{Tension in R₁ is} \quad Tac = 150 \text{ N} \]

Tipping or No Tipping?

\[ \Sigma M_o = 0 \quad \rightarrow \quad (Na)(d) - (Tac)(0.3) = 0 \quad \rightarrow \quad d = 0.075 \text{ m} < 1.6/2 = 0.8 \text{ m} \quad \text{no tipping.} \]

Will block C tip or slip (justify fully your answer)?

\[ \Sigma F_y = 0 \]
\[ \text{Nc} - (W_c) \cos(30^\circ) = \text{Nc} - 200 \cos(30^\circ) = 0 \rightarrow \]
\[ \text{Nc} = 173.2051 \text{ N} \]
\[ F_c = \mu \text{ Nc} = 0.25 \text{ Nc} = 43.3013 \text{ N} \]

\[ \Sigma F_x = 0 \]
\[ T_{cd} - T_{ac} - F_c W_c \sin(30^\circ) = T_{cd} - 150 - 43.3013 - 200 \sin(30^\circ) = 0 \rightarrow T_{cd} = 293.3013 \text{ N} \]

\[ \Sigma M_p = 0 \]
\[ (T_{ac})(0.5) + (W_c \cos(30^\circ))(0.2) + (W_c \sin(30^\circ))(0.5) - (T_{cd})(1.0) = 0 \]
\[ (150)(0.5) + (200 \cos(30^\circ))(0.2) + (200 \sin(30^\circ))(0.5) - (T_{cd})(1.0) = 0 \]
\[ \rightarrow T_{cd} = 159.6410 \text{ N} \]

159.6410 N < 293.3013 N \rightarrow \text{Tipping}

What is the tension in rope R2 between block C and peg D?
Tension in R2 is 159.6410 N

What is the weight \( W_E \) of block E?

\[ T_1 = 159.6410 \]
\[ T_2 = T_1 e^{\mu \beta} = 159.6410 \ e^{(0.3)(2\pi/3)} = 299.2400 \text{ N} \]
\[ \rightarrow W_E = 299.2400 \text{ N} \]
Problem 4 (20 points)

The magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ are $F_1 = 300 \text{ N}$ and $F_2 = 500 \text{ N}$, respectively. For $\mathbf{F}_1$, $\alpha = 120^\circ$ and $\gamma = 30^\circ$. For $\mathbf{F}_2$, $\phi = 45^\circ$ and $\theta = 60^\circ$. Determine:

(a) The vector form of the resultant $\mathbf{R}$ of $\mathbf{F}_1$ and $\mathbf{F}_2$  
(b) The magnitude of the resultant $R$  
(c) The direction cosines of the resultant  
(d) The angle between $\mathbf{F}_1$ and $\mathbf{F}_2$

\[ \begin{array}{|l|l|}
\hline
\text{Value} & \text{Unit} \\
\hline
(a) & \mathbf{R} = 26.8\mathbf{i} + 306\mathbf{j} + 612\mathbf{k} \text{ N} \\
(b) & R = 684 \text{ N} \\
(c) & \cos \alpha = 0.039, \cos \beta = 0.447, \cos \gamma = 0.894 \\
(d) & \text{The angle between } \mathbf{F}_1 \text{ and } \mathbf{F}_2 = 64.2^\circ \\
\hline
\end{array} \]

Note: You need to show all work to receive full credit.

Solution:

(a)

\[ \mathbf{F}_1 = 300(\cos 120^\circ \mathbf{i} + \sqrt{1 - \cos^2 120^\circ} - \cos 30^\circ \mathbf{j} + \cos 30^\circ \mathbf{k}) \]
\[ = 300(-0.5\mathbf{i} + 0\mathbf{j} + 0.866\mathbf{k}) = -150\mathbf{i} + 258\mathbf{k} \text{ N} \]

\[ \mathbf{F}_2 = 500(\cos 45^\circ \mathbf{i} + \cos 45^\circ \mathbf{j} + \sin 45^\circ \mathbf{k}) \]
\[ = 500(0.3535\mathbf{i} + 0.6123\mathbf{j} + 0.707\mathbf{k}) = 176.75\mathbf{i} + 306.14\mathbf{j} + 353.5\mathbf{k} \text{ N} \]

\[ \mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = -150\mathbf{i} + 258\mathbf{k} + 176.75\mathbf{i} + 306.14\mathbf{j} + 353.5\mathbf{k} = 26.8\mathbf{i} + 306\mathbf{j} + 612\mathbf{k} \text{ N} \]

(b) \[ R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{26.8^2 + 306^2 + 612^2} = 684 \text{ N} \]

(c) Each of the following is 2 points:

\[ \cos \alpha = \frac{R_x}{R} = \frac{26.8}{684} = 0.039 \]

\[ \cos \beta = \frac{R_y}{R} = \frac{306}{684} = 0.447 \]

\[ \cos \gamma = \frac{R_z}{R} = \frac{612}{684} = 0.894 \]

(d)

\[ \cos \theta_{12} = \frac{\mathbf{F}_1 \cdot \mathbf{F}_2}{F_1 F_2} = u_{1x} u_{2x} + u_{1y} u_{2y} + u_{1z} u_{2z} \]
\[ = (-0.5)(0.3535) + 0(0.6123) + 0.866(0.707) = 0.4355 \]

\[ \theta_{12} = \cos^{-1}(0.4355) = 64.2^\circ \]
Problem 5 (20 points)

A boom \( OA \), supporting a load of 500N, is held in place by two cables \( AB \) and \( AC \) as shown.

a) Draw a complete and separate FBD of the particle  

b) Express all the forces in the FBD in Cartesian Vector Form  

c) Write the equations of equilibrium for that particle  

d) Solve the equations and determine the tension force in cables \( AB \) and \( AC \) as well as the force in \( OA \).

\[
\begin{align*}
\tau_B &= \frac{T_B}{3} \left( -2\hat{i} + \hat{j} - 2\hat{k} \right) \text{N} \quad \vec{u}_{TB} = \frac{(-6,3,-6)}{9} \\
\tau_C &= \frac{T_C}{7} \left( -2\hat{i} + 3\hat{j} - 6\hat{k} \right) \text{N} \quad \vec{u}_{TC} = \frac{(-2,3,-6)}{7} \\
\vec{F}_{OA} &= \frac{F_{OA}}{13} \left( 4\hat{i} - 3\hat{j} + 12\hat{k} \right) \text{N} \quad \vec{u}_{F_{OA}} = \frac{(2,-1.5,6)}{6.5} \\
\vec{W} &= -500\hat{k} \text{N} \\
\sum F_x &= 0 \rightarrow -\frac{2}{3}T_B - \frac{2}{7}T_C + \frac{4}{13}F_{OA} = 0 \\
\sum F_y &= 0 \rightarrow \frac{1}{3}T_B + \frac{3}{7}T_C - \frac{3}{13}F_{OA} = 0 \\
\sum F_z &= 0 \rightarrow -\frac{2}{3}T_B - \frac{6}{7}T_C + \frac{12}{13}F_{OA} = 500
\end{align*}
\]

d) Solving the 3 equations we get:

\( T_B = 368 \text{ N} \) \\
\( T_C = 286 \text{ N} \) \\
\( F_{OA} = 1,062 \text{ N} \) (C)
Problem 6 (20 points)

For the truss shown, find

(a) The zero-force members, if any (2)
(b) The support reactions (4)
(c) The forces in members CD, DE, CB, and CE using the method of joints (6)
(d) The forces in members BE, BF, and FG using the method of sections (use only one section) (8)

NOTE: Draw a complete free body diagram for each question and state whether the forces in each required member is tension or compression.

Solution

a) Zero force members → BF

b) \( \sum F_x = 0 \)
\[
5 + 10 + 10 + A_x = 0 \\
A_x = -25 \text{ kN}
\]
\( 25 \text{ kN} \) (→)

c) \( \sum M_G = 0 \)
\[
-A_y + 4 \cdot 10 + 4 \cdot 10 \cdot k_B - 5 \cdot 12 = 0 \\
A_y = -45 \text{ kN}
\]
\( 45 \text{ kN} \) (↑)

d) \( \sum F_y = 0 \)
\[
-45 + G_y - 5 = 0 \\
G_y = 50 \text{ kN} \uparrow
$\sum F_x = 0 \quad 5 + DE \sin 45^\circ = 0$
\[ DE = -7.07 \text{ kN} \]
\[ 7.07 \text{ kN} \quad (\text{c}) \]

$\sum F_y = 0$
\[ -DC - DE \cos 45^\circ = 0 \]
\[ DC = 5 \text{ kN} \quad (\text{T}) \]

$\sum F_y = 0$
\[ DC = CB = 5 \text{ kN} \quad (\text{T}) \]

$\sum F_x = 0$
\[ 10 + CE = 0 \]
\[ CE = -10 \text{ kN} \]
\[ 10 \text{ kN} \quad (\text{c}) \]

$FB = 0 \quad (\text{zero force member})$

$CB = 5 \text{ kN} \quad (\text{from c})$

$\sum M_B = 0$
\[ -5 \times 8 - 10 \times 4 - 5 \times 4 - FG \times 4 = 0 \]
\[ FG = -25 \text{ kN} \quad [25 \text{ kN} \quad (\text{c})] \]

$\sum F_y = 0$
\[ -FG - EB \cos 45^\circ - CB = 0 \]
\[ EB = \frac{CB - 5 - FG}{\cos 45^\circ} = 21.2 \text{ kN} \quad (\text{T}) \]