This exam has two parts. Part I has 20 multiple choice questions, worth two points each. Part II consists of six relatively short problems, worth ten points each. The short problems can be worked out on the front page of the sheet provided, but use the back if you need more room. In any case please be neat!

Also, two extra pages are provided at the back. Use these if you need to, but be sure to indicate which problem you are working on these pages.

You may use your textbook, course notes, or any other reference you may have other than another human. You are welcome to use your calculator or computer, although the test is designed so that these are not necessary.

Good luck!

Part I (Total): 

Part II
Problem 1: 
Problem 2: 
Problem 3: 
Problem 4: 
Problem 5: 
Problem 6: 

Total: 
Part I: Multiple Choice Questions (Two points each)

1. A beam sends 100,000 particles at a target with $10^{20}$ scattering centers per cm$^2$. A 1 cm$^2$ detector sits 1 m away, at an angle such that the differential cross section is $10^{-20}$ cm$^2$ per steradian. The number of detector particles is approximately

   A. 10.
   B. 100.
   C. 1000.
   D. 10,000.
   E. 100,000.

2. Given two relativistic four-momenta $p_1$ and $p_2$, which of the following will generally have different values in different reference frames?

   A. $p_1$
   B. $p_1^2$
   C. $p_1 \cdot p_2$
   D. $(p_1 + p_2)^2$
   E. $(p_1 - p_2)^2$

3. A simple harmonic oscillator consists of a mass $m$ connected to a spring with stiffness $k$. If the stiffness is suddenly doubled, the oscillation period is multiplied by

   A. $1/2$
   B. $1/\sqrt{2}$
   C. 1
   D. $\sqrt{2}$
   E. 2

4. A plumb bob hanging near the Earth’s surface does not point exactly at the Earth’s center of mass. If the Earth was completely isotropic,

   A. this would no longer be true.
   B. this would still be true because the Earth is non-spherical.
   C. this would still be true because the Earth is in orbit about the Sun.
   D. this would still be true because of the Coriolis force of the rotating Earth.
   E. this would still be true because of the centrifugal force of the rotating Earth.
5. For a system with \( n \) degrees of freedom, we can define a \( 2n \)-dimensional “space” out of the generalized coordinates and their conjugate momenta. We proved a theorem about the “volume” in this space. In particular, we found that this volume

A. is always zero.
B. can be written in terms of a strain tensor.
C. must remain constant as the system evolves.
D. undergoes oscillations about its principal axes.
E. has a “center of mass” that moves with constant velocity.

6. A mass is connected to a spring and moves through a viscous medium with linear damping. If the mass is \( m \) and the spring has stiffness \( k \), what value of the damping coefficient makes any oscillations die out as quickly as possible?

A. \( km \)
B. \( 2km \)
C. \( \sqrt{km} \)
D. \( \sqrt{2km} \)
E. \( 2\sqrt{km} \)

7. Two identical masses \( m \) are attached to the ends of a massless rod of length \( 2\ell \). The principal moments of inertia about the center of mass are

A. 0, 0, and \( m\ell^2 \)
B. 0, \( m\ell^2 \), and \( m\ell^2 \)
C. 0, \( 2m\ell^2 \), and \( 2m\ell^2 \)
D. 0, \( 4m\ell^2 \), and \( 4m\ell^2 \)
E. \( m\ell^2 \), \( 2m\ell^2 \), and \( 4m\ell^2 \)

8. The figure below shows a pendulum (of mass \( m \)) suspended from the roof of a railroad car moving to the right (the \( x \)-direction) with constant acceleration \( A \).

If \( T \) is the tension in the pendulum string, how would an observer in the railroad car write Newton’s Second Law of Motion for the \( x \)-component?

A. \( m\ddot{x} = -T \sin \phi \)
B. \( m\ddot{x} = -T \sin \phi + mA \)
C. \( m\ddot{x} = -T \sin \phi - mA \)
D. \( m\ddot{x} = -T \sin \phi - mg + mA \)
E. \( m\ddot{x} = -T \sin \phi - mg - mA \)
9. A damped oscillator is made from a spring with stiffness constant 5 N/m and a 10 kg mass. The damping force is $-bv$ where $b = 4$ N·sec/m. How long does it take for the amplitude of the oscillator to decrease by a factor of $1/e$?

A. 1/2 sec  
B. 1 sec  
C. 2 sec  
D. 5 sec  
E. 10 sec

10. Suppose you need to transfer a spacecraft from one circular orbit about the sun to another, farther orbit about the sun. You do this by firing only short bursts of rockets to speed up or slow down the spacecraft in the direction it is already moving. Which of the following is correct?

A. Speed up at the inner orbit and speed up at the outer orbit.  
B. Speed up at the inner orbit and slow down at the outer orbit.  
C. Slow down at the inner orbit and speed up at the outer orbit.  
D. Slow down at the inner orbit and slow down at the outer orbit.  
E. Speed up at the inner orbit, slow down at the outer orbit, and speed up in between.

11. The Atwood’s Machine below is made from two masses $m_1$ and $m_2$ connected by a massless string of fixed length around a massless pulley:

The coordinate $x$ measures the downward position of $m_1$ as shown. The Lagrangian is

A. $\mathcal{L} = \frac{1}{2}(m_1 + m_2)\dot{x}^2 - (m_1 + m_2)gx$  
B. $\mathcal{L} = \frac{1}{2}(m_1 + m_2)\dot{x}^2 + (m_1 + m_2)gx$  
C. $\mathcal{L} = \frac{1}{2}(m_1 + m_2)\dot{x}^2 + (m_1 - m_2)gx$  
D. $\mathcal{L} = \frac{1}{2}(m_1 - m_2)\dot{x}^2 - (m_1 + m_2)gx$  
E. $\mathcal{L} = \frac{1}{2}(m_1 - m_2)\dot{x}^2 - (m_1 - m_2)gx$

12. What of the following conditions is sufficient for the Hamiltonian to be a constant of the motion?

A. No external forces act on the system.  
B. No external torques act on the system.  
C. The Hamiltonian equals the mechanical energy.  
D. The Lagrangian does not depend explicitly on time.  
E. The Hamiltonian does not depend explicitly on time.
13. The Lagrangian for a mechanical system is
\[ \mathcal{L}(q, \dot{q}) = a\dot{q}^2 + b\dot{q}^4 \]
where \( q \) is a generalized coordinate and \( a \) and \( b \) are constants. The momentum conjugate to \( q \) is
A. \( bq \)
B. \( a\dot{q} \)
C. \( a\ddot{q} \)
D. \( 2a\dot{q} \)
E. \( 4b\dot{q}^3 \)

14. An axially symmetric (i.e. \( \lambda_1 = \lambda_2 \) in terms of your textbook) object rotates freely. At \( t = 0 \) the rotation is purely about an axis perpendicular to the principal axis. Which of the following statements is correct?
A. The object eventually stops rotating.
B. The angular velocity vector \( \omega \) never changes.
C. The object exhibits a motion known as nutation.
D. The axis of rotation precesses about the principal axis.
E. The object gyrates until all of the rotation is about the principal axis.

15. Which coordinate (or combination) is “ignorable” for the following Lagrangian?
\[ \mathcal{L}(x, y, \dot{x}, \dot{y}) = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) - ax \]
A. \( x \)
B. \( y \)
C. \( x + y \)
D. \( x - y \)
E. \( x^2 + y^2 \)

16. A mass is connected to a spring and moves through a viscous medium so that there is only very slight damping. This oscillator has a certain resonant frequency \( \omega \) and \( Q \)-value. To increase \( Q \) while keeping \( \omega \) unchanged, you need to
A. increase the damping and increase the spring constant.
B. increase the damping and decrease the spring constant.
C. decrease the damping and increase the spring constant.
D. increase the damping but not change the spring constant.
E. decrease the damping but not change the spring constant.
17. An system undergoes unforced, damped oscillations at a frequency $\omega_0$. It is subjected to a sinusoidal driving force with frequency $\omega$ which continues indefinitely. After a long time has gone by, the system oscillates at frequency

A. $\omega$
B. $\omega_0$
C. $\sqrt{\omega \omega_0}$
D. $\omega_0 + 2\omega$
E. $(\omega + \omega_0)/2$

18. Which of the following does not describe a conservative force field?

A. $F(x, y) = x\hat{i} + y\hat{j}$
B. $F(x, y) = x\hat{i} - y\hat{j}$
C. $F(x, y) = y\hat{i} - x\hat{j}$
D. $F(x, y) = y\hat{i} + x\hat{j}$
E. $F(x, y) = x^2\hat{i} + y^2\hat{j}$

19. The moment of inertia tensor for some object is, in some coordinate system,

$$I = ML^2 \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 9 \end{bmatrix}$$

The angular velocity is $\omega = \omega_0(\hat{e}_1 + \hat{e}_3)$ in this coordinate system. The kinetic energy is

A. $ML^2 \omega_0^2$
B. $2ML^2 \omega_0^2$
C. $4ML^2 \omega_0^2$
D. $8ML^2 \omega_0^2$
E. $16ML^2 \omega_0^2$

20. Your instructor and teaching assistant for this course are

A. Lasagna and Stick
B. Orzo and Rigatoni
C. Ronzoni and Barilla
D. Godzilla and Mothra
E. Lanzillo and Napolitano
Part II: Short Problems (10 points each)

**Problem 1.** A student sits on a stationary sled with a pile of snowballs. The entire mass is $M$ and the snowballs each have mass $m \ll M$. The sled slides without friction. The student throws snowballs horizontally off the sled at a speed $v_0$ relative to herself. If eight snowballs are thrown in the same direction and at one-second intervals, what is the final speed of the sled, assuming that you can neglect the change in mass of the sled+student+snowballs?
Problem 2. An inviscid fluid flows steadily in the (horizontal) $x$-direction in a region $x > 0$ with velocity field $\mathbf{v} = v_0(x/a)\hat{i}$. Determine how the density $\rho(x)$ and the pressure $p(x)$ vary with $x$. Clearly indicate any integration constants you need to introduce.
Problem 3. An object with mass $m$ is launched vertically upward from the ground with an initial speed $v_0$. It is acted on by a quadratic drag force $cv^2$. Find the time it takes to reach its highest point, in terms of $m$, $g$, $c$, and $v_0$. 
Problem 4. Five questions: The figure shows a bead of mass \( m \) constrained to lie on a long, straight wire. The wire is constrained to rotate in a horizontal plane about the origin with angular speed \( \omega \). Find the Lagrangian \( \mathcal{L}(r, \dot{r}) \) where \( r \) measures the distance of the bead from the origin. Find the conjugate momentum \( p_r \) and the Hamiltonian \( \mathcal{H}(r, p_r) \). If the bead starts at rest (along the wire) at a position \( r = a \), find the position \( r(t) \) as a function of time. How does the bead’s energy change with time?
Problem 5. An axially symmetric space station floats freely in space. Rockets can provide a constant torque \( \Gamma \) about the symmetry axis, with no torque about the other axes. The symmetry axis lies in the 3-direction. Solve Euler’s equations for the angular velocity vector \( \mathbf{\omega}(t) \) (in the body axis) subject to the initial conditions \( \mathbf{\omega}(0) = \alpha \hat{e}_1 \). [Hint: Define \( \eta(t) \equiv \omega_1(t) + i\omega_2(t) \) to solve your coupled equations for \( \omega_1(t) \) and \( \omega_2(t) \).] Put your answer in terms of \( \lambda_1 = \lambda_2 \equiv \lambda_0, \lambda_3, \Gamma, \) and \( \alpha \). Briefly describe the motion for the cases \( \alpha = 0 \) and \( \alpha \neq 0 \).
Problem 6. A thin hoop of radius $R$ and mass $M$ oscillates in its own plane hanging from a single fixed point. Attached to the hoop is a small bead with mass $M$ which is constrained to move, without friction, along the hoop. Find the two normal mode frequencies of this contraption, in terms of $R$, $M$, and $g$, assuming small oscillations. [Hints: (1) Measure all angles relative to the (downward) vertical; (2) The kinetic energy of the hoop is easy to write down using “$I\dot{\theta}^2/2$” and the parallel axis theorem.]
Extra Paper #1: Please show the problem on which you are working!
Extra Paper #2: Please show the problem on which you are working!