This exam has four questions and you are to work all of them. You must hand in your paper by the end of class time (9:50pm) unless prior arrangements have already been made with the instructor.

Note that not all of the problems are worth the same number of points.

You may use your textbook, course notes, or any other reference you may have other than another human. You are welcome to use your calculator or computer, although the test is designed so that these are not absolutely necessary.

Good luck!

Problem 1: 

Problem 2: 

Problem 3: 

Problem 4: 

Total:
Problem 1 (20 points). An object with mass $m_1$ moves relativistically with speed $v$. It collides with a stationary object of mass $m_2$ and the two masses stick together. Find the (rest) mass of the the combined object, and show that you get the right answer for $v \ll c$. 
Problem 2 (30 points). The figure shows two equal masses \( m \) at the ends of a massless rod of length \( 2\ell \), rotating with angular velocity \( \omega \) about an axis which passes through the center of mass. The normal vector to the rod makes an angle \( \alpha \) with respect to the axis of rotation. At the instant shown, the rod lies in the \( xz \) plane. Use this coordinate system, at this instant, for the calculations below.

a. (10 points) Find all nine components of the inertia tensor for this coordinate system.

b. (5 points) Find the (vector) angular momentum for the configuration as shown.

c. (5 points) Find the kinetic energy for the configuration as shown.

d. (10 points) Calculate the principal moments of inertia. Also give the principal axis directions; you don’t need to work out all the math, but in any case, explain physically why your principal moments and axes are correct.
Extra paper for Problem 2.
Problem 3 (25 points). An object falls from rest, from a height $h \ll R$ above the Earth’s surface, where $R$ is the radius of the Earth, at colatitude $\theta$. The Coriolis force deflects the object, and it hits the ground some distance away from where a plumb bob would hang. If $\Omega$ is the rotational angular velocity of the Earth, find the lowest nonzero order approximation to the north/south component of the deflection on the ground. Express your result in terms of $h$, $g$, $R$, $\Omega$, and $\theta$. Is the deflection to the north or south in the Northern Hemisphere?
Problem 4 (25 points). The plot gives recent data taken with the Keck telescope in Hawaii, showing the position of a star, at different times, orbiting the center of our Galaxy. Distance are given in AU, relative to an arbitrary origin, where 1 AU is the distance from the Earth to the Sun. The time it takes for one complete orbit is observed to be 16 years.

Use this data to determine, as accurately as you can from the plot, the mass of the object at the center of the Galaxy. (Assume the orbit lies in the plane of the page.) Express your answer as a factor times the mass of the Sun.