Physics I Math Assessment

The purpose of the following 10 questions is to assess some math skills that you will need in Physics I. These questions will help you identify some math areas that you may want to review as you take Physics I. As always, select the best answer from the choices given.

1. Consider the following equation:

\[ \frac{a}{(d - x)^2} = \frac{b}{x^2} \]

where \( a > 0, b > 0, \) and \( d > 0. \) We are looking for a solution for \( x \) where \( 0 < x < d. \)

A. It is impossible to solve this equation except with numerical techniques because it involves the reciprocals of polynomials and there is no general formula for that.
B. There is exactly one solution and it can be found using a linear equation.
C. There are exactly two solutions that are found using the quadratic formula.
D. It is possible to write an expression for the solution(s) using the quadratic formula, but it is impossible to know which one(s) (if any) satisfy the condition on \( x \) without knowing more about \( a, b, \) and/or \( d. \)
E. None of the above.

In questions 2-4, the function \( f(x) \) is continuous and differentiable on the interval \([a,b]\). That means that the graph of \( f(x) \) is a continuous curve with no breaks and the slope of \( f(x) \) is well-defined. When we speak of the slope of \( f(x) \), we mean the slope of the graph of \( f(x) \) versus \( x. \)

2. If the slope of \( f(x) \) is negative in \([a,b]\), then

A. \( f(x) \leq 0 \) at all points in \([a,b]\).
B. \( f(x) = 0 \) at some point in \([a,b]\).
C. \( f(a) > f(b) \).
D. All of the above.
E. None of the above.

3. If \( f(x) < 0 \) in \([a,b]\), then the slope of \( f(x) \)

A. is always positive at all points in \((a,b)\).
B. is always negative at all points in \((a,b)\).
C. must be zero at some point in \([a,b]\).
D. All of the above.
E. None of the above.
4. In the interval \([a,b]\), the slope of \(f(x)\) is constant. The average value of \(f(x)\) in that interval is

A. \(\frac{1}{2}[f(a) + f(b)]\).
B. \(f(\frac{1}{2}(a + b))\).
C. the area under the graph of \(f(x)\) between \(a\) and \(b\), divided by \((b-a)\).
D. All of the above.
E. None of the above.

5. In the graph below (not necessarily to scale), \(f(x)\) is defined in the interval \([a,b]\) by the straight line segments connecting the points as shown.

The area under the graph

A. is always positive or zero.
B. is zero if \(p = q\).
C. can be calculated using area formulas for triangles and rectangles.
D. All of the above.
E. None of the above.
6. The vector \( \vec{v} \) shown below has length \( c \). Its angle measured counter-clockwise from horizontal axis is \( \theta \) and its angle measured clockwise from the vertical axis is \( \phi \).

![Diagram of vector v](image)

The vertical component of \( \vec{v} \) is given by

A. \( c \sin(\theta) \).
B. \( c \cos(90^\circ - \theta) \).
C. \( c \cos(\phi) \).
D. All of the above.
E. None of the above.

7. In the diagram below, the vectors \( \vec{u} \) and \( \vec{v} \) are at right angles to each other. The length of \( \vec{u} \) is \( c \) and the length of \( \vec{v} \) is \( d \). The horizontal and vertical components of \( \vec{u} \) are \( a \) and \( b \) respectively.

![Diagram of vectors u and v](image)

The vertical component of \( \vec{v} \), \( e \), is

A. \( a \ d / c \).
B. \( b \ d / c \).
C. \( b \ d / a \).
D. \( \sqrt{d^2 - c^2} \).
E. None of the above.
8. Consider two vectors, \( \vec{u} \) and \( \vec{v} \), and their sum, \( \vec{w} = \vec{u} + \vec{v} \). Which orientation of the vectors shown below gives the smallest magnitude (length) of \( \vec{w} \)? (The magnitudes of \( \vec{u} \) and \( \vec{v} \) don’t change in the diagrams, but don’t assume that they are drawn to scale.)

   A. \[ \begin{align*}
   &\quad \vec{u} \\
   &\quad \vec{v}
   \end{align*} \]

   B. \[ \begin{align*}
   &\quad \vec{u} \\
   &\quad \vec{v}
   \end{align*} \]

   C. \[ \begin{align*}
   &\quad \vec{u} \\
   &\quad \vec{v}
   \end{align*} \]

   D. It doesn’t make any difference since the magnitudes of \( \vec{u} \) and \( \vec{v} \) don’t change.

   E. There is no way to tell without knowing the magnitudes of \( \vec{u} \) and \( \vec{v} \).

9. (Challenge problem.) The vectors \( \vec{u} \) and \( \vec{v} \) are given as shown below, with lengths \( c \) and \( d \) respectively. The angle of \( \vec{u} \) with respect to the reference line is \( \theta \) and the angle of \( \vec{v} \) with respect to the reference line is \( \phi \), going counter-clockwise.

   \[ \begin{align*}
   &\quad \vec{v} \\
   &\quad \vec{u}
   \end{align*} \]

   The magnitude (length) of the sum, \( \vec{w} = \vec{u} + \vec{v} \), is

   A. \( (c + d) \cos(\phi - \theta) \).

   B. \( c \cos(\theta) + d \cos(\phi) \).

   C. \( \sqrt{c^2 \cos^2(\theta) + d^2 \cos^2(\phi)} \).

   D. \( \sqrt{c^2 + d^2 + 2cd \cos(\theta - \phi)} \).

   E. The magnitude of \( \vec{w} \) cannot be determined unless the direction of the reference line is known.
10. Dick and Jane went into business selling mud pies. They set up two mud pie stands at opposite ends of the neighborhood to maximize their potential customers. On Monday, they sold a total of 100 pies. One Tuesday, Dick took the day off because his sales were so good on Monday. Jane figured that she could triple her sales from Monday if she copied Dick’s slick techniques. Unfortunately, Dick’s customers from Monday all returned their pies to Jane on Tuesday. Jane met her Tuesday sales target, but she only sold 20 new pies after reselling all of Dick’s returned pies from Monday. What equations would you use to determine how many pies Jane sold on Monday?

A. \( J + D = 100 \)
   \( 3J = 120 \)
B. \( J + D = 100 \)
   \( 3J - D = 20 \)
C. \( J + D = 100 \)
   \( 3J + D = 120 \)
D. There is not enough information to determine a unique answer.
E. There is contradictory information so there is no answer.