(1) For the following horizontal system of two masses and three springs (Taylor Figure 11.1), study the motion for $k_1 = k_2 = k_3 \equiv k$, with $m_1 = m$ and $m_2 = 10m \gg m_1$. (You are welcome to appropriately modify the MATHEMATICA notebook that I have posted on the course web page.) With $k/m = 4\pi^2$, plot the motions $x_1(t)$ and $x_2(t)$ for initial conditions $\dot{x}_1(0) = \dot{x}_2(0) = 0$ and (a) $x_1(0) = 1$ and $x_2(0) = 0$, and (b) $x_1(0) = 0$ and $x_2(0) = 1$. (These two cases give very different results.) Explain, using basic Newtonian mechanics, why the two plots have such a qualitative difference.

(2) (See Taylor 11.25.) Consider a system of horizontal masses and springs such as Figure 11.1 (above), but with three identical masses $m$ and four identical springs $k$. (a) Find the three normal mode frequencies. (b) Find the eigenvectors for the three modes, and describe their motion. (c) Find the normal coordinates $\xi^{(i)}$ for modes $i = 1, 2, 3$.

(3) (See Taylor 11.26.) A bead of mass $m$ is threaded on a frictionless circular wire hoop of radius $R$ and mass $m$. The hoop is suspended at point $A$ and is free to swing in its own vertical plane, as shown at the right. Using $\phi_1$ and $\phi_2$ as generalized coordinates, solve for the normal mode frequencies, assuming small oscillations, and find the normal mode eigenvectors. Describe the motion to which they correspond, and find the normal coordinates. Note that the kinetic energy of the hoop is $\frac{1}{2}I\dot{\phi}_1^2$ where $I$ is the moment of inertia about the point $A$; recall the parallel axis theorem. Also, the potential energy of the hoop (due to gravity) is determined as if all the mass were concentrated at its center of mass.

(4) (See Taylor 16.2.) Below are Taylor Figures 16.1(a), 16.1(b), and 16.20:

Obtain the equation of motion for a continuous string under tension $T$ using Newtonian mechanics by taking the limit of a series of $n$ connected masses as $n \to \infty$. In this limit, the spacing $b \to 0$ and mass $m \to 0$ in such a way that $\mu = m/b$ remains constant. So, set $m = \mu b$ and write down Newton’s Second Law for the position $u_i$ of the $i$th mass and show that it goes over to the wave equation as $b \to 0$. The vertical displacement is always “small” so that tension vector $\mathbf{T}$ only has small angles with respect to the horizontal (equilibrium) line of the string.