(1) A particle of mass $m$ moves in one dimension $x$. It is subject to a force $F(t) = F_0 e^{-t/\tau}$, where $\tau$ is a positive constant, and starts from rest at $x = 0$ and $t = 0$. Find the velocity $v(t) = \dot{x}(t)$ and position $x(t)$ as functions of time. Also find the velocity $v(t)$ for times $t \gg \tau$.

(2) A particle of mass $m$ moves in two dimensions according to plane polar coordinates $r$ and $\phi$. It is acted on by a force $\mathbf{F} = \hat{r} F_r(r, \phi) + \hat{\phi} F_\phi(r, \phi)$. Construct the particle angular momentum $\ell = |r \times (mv)| = m|\mathbf{r} \times \dot{\mathbf{r}}|$ in terms of $m$, $r$, $\dot{r}$, $\phi$, and $\dot{\phi}$. Calculate its time derivative $d\ell/dt$, and show that $\ell$ is a constant of the motion if $F_\phi = 0$.

(3) A particle of mass $m$ moves in one dimension according to the potential energy function

$$U(x) = -U_0 \frac{a^2}{a^2 + x^2}$$

where $U_0$ and $a$ are positive constants. It has a total mechanical energy $-U_0 < E < 0$. Sketch the potential energy as a function of $x$ and show (analytically) that $x = 0$ is a point of stable equilibrium. Find the “classical turning points” $x_m$, that is the maximum and minimum values of $x$, in terms of $E$, $a$, $m$, and $U_0$. For $E + U_0 \ll U_0$, show that $|x_m| \ll a$. (What does this condition mean, physically?) Find the (angular) frequency and also the period of oscillation for $E + U_0 \ll U_0$.

(4) Continue problem (3) using MATHEMATICA, MATLAB, or some other computer program, and numerically determine the period as a function of the (dimensionless) variable $y_m \equiv x_m/a$. It is easiest to write the period $T$ as a definite integral over one quarter of the period, and then multiply by four. Your computer can do the integral numerically. Make a plot of $T$ versus $y_m$ and show that you get the correct result (from problem 3) as $y_m \to 0$.

(5) For each of the following force fields $\mathbf{F}(\mathbf{r})$, prove which are conservative and which are not. For the conservative forces, find the corresponding potential energy $U(\mathbf{r})$, and verify by direct differentiation that $\mathbf{F} = -\nabla U$. The parameter $k$ is a constant, independent of $\mathbf{r}$. You are welcome to make use of vector calculus results in different coordinates systems, such as those listed on the inside back cover of Taylor.

- $\mathbf{F} = k\mathbf{r} = k(x\hat{x} + y\hat{y} + z\hat{z})$
- $\mathbf{F} = k(x\hat{x} + 2y\hat{y} + 3z\hat{z})$
- $\mathbf{F} = k(y\hat{x} + x\hat{y})$
- $\mathbf{F} = k(y\hat{x} - x\hat{y})$
- $\mathbf{F} = f'(r)\hat{r}$ for an arbitrary function $f(r)$; $f'(r)$ is the derivative.