This exam has four questions and you are to work all of them. You must hand in your paper by the end of class time (5:50pm) unless prior arrangements have already been made with the instructor.

Note that not all of the problems are worth the same number of points.

You may use your textbook, course notes, or any other reference you may have other than another human. You are welcome to use your calculator or computer, although the test is designed so that these are not absolutely necessary.

Good luck!

Problem 1: ______________________

Problem 2: ______________________

Problem 3: ______________________

Problem 4: ______________________

Total: ______________________
Problem 1 (25 points). An object with mass $m$ moves in one dimension $x$ according to a potential energy $U(x) = U_0(x^3 - 3x + 2)$ where $U_0$ is a constant.

a. (10 points) Find $x$ for all equilibrium points and identify them as stable or unstable.

b. (15 points) For all points of stable equilibrium, find the (angular) frequency of small amplitude oscillations about the equilibrium point.
Problem 2 (25 points). A particle with mass $m$ moves in one dimension $x$. It is subject to a time-dependent force $F(t) = f_0 e^{-bt}$, with $b > 0$. It starts from rest at $x = 0$. Find its (a) velocity $v(t)$, (b) position $x(t)$, and (c) velocity after a long time, that is $v(t \to \infty)$.
Problem 3 (20 points). Two masses move horizontally without friction as shown:

![Diagram of masses and springs](image)

The larger mass ($3m$) is connected by a spring to a fixed wall, and also to the smaller mass ($2m$). The smaller mass is connected *only* to the larger mass. The springs are identical, each with stiffness $k$. Find the eigenfrequencies for this system, and quantitatively describe the amplitudes and motions of the corresponding modes.
Problem 4 (30 points). Two identical masses $m$ are connected by a string of fixed length. The string passes through a hole in a horizontal, flat, frictionless table, so that one mass is constrained to the table surface and the other mass hangs vertically underneath.

a. (10 points) Write the Lagrangian in terms of the polar coordinates $r$ and $\phi$ that locate the mass on the table surface. Note: As $r$ increases, the mass under the table rises.

b. (10 points) Find (differential) equations of motion for $r(t)$ and $\phi(t)$. (Don’t solve them.)

c. (10 points) Show that $\phi$ is an “ignorable coordinate” and identify the associated constant of the motion. Use this constant and (b) above to write a differential equation that just involves $r$ and not $\phi$. 