PHYS4330 Theoretical Mechanics Fall 2011 Course Summary

Concepts in Mathematics


The calculus of variations and its application to physical problems.

Mechanics according to Newton’s Second Law of Motion

\( \mathbf{F} = m \mathbf{a} = m \ddot{\mathbf{r}} \) as a differential equation to be solved for \( \mathbf{r}(t) \). Expressions in Cartesian, cylindrical, and spherical coordinates. Conservative forces defined through path independence and Stokes’ Theorem. Kinetic and potential energy. Total energy as a first integral of the motion. Special properties of “central” forces, including conservation of (particle) angular momentum as a first integral of the motion. Fictitious forces in non-inertial reference frames, with special consideration of rotating reference frames and the surface of the Earth.

Mechanics according to Hamilton’s Principle

Statement of Hamilton’s Principle in terms of the action. The Lagrangian function \( L(q, \dot{q}, t) \) and the derivation of Lagrange’s Equations. Derivation of \( \mathbf{F} = m \mathbf{a} = m \ddot{\mathbf{r}} \). Extensions to generalized coordinates with arbitrary number of degrees of freedom. Conservation laws as seen through Lagrange’s equations. Application of constraints through Lagrange multipliers. Examples of mechanical systems solved with Lagrange’s equations.

The Hamiltonian function \( H(q, p) \) and Hamilton’s equations of motion. Ignorable coordinates and first integrals of the motion. Phase space and Liouville’s Theorem.

Dynamics of Multiparticle Systems

External and internal forces. Definition and dynamics of the center of mass. Conservation of total momentum and total angular momentum following “Newton’s Third Law.” Separation of motion of and relative to the center of mass. Moment of inertia and torque.

Orbital Mechanics and Kepler’s Laws

The two-body problem and reduced mass. Application to central force problems. Motion described through “effective potentials” incorporating the “angular momentum barrier.” Motion described through a differential equation for “orbits.” Newton’s law of universal gravitation and the derivation of Kepler’s three laws of planetary motion. Orbits as conical sections for motion in \( 1/r \) potentials.
Oscillations with One or More Degrees of Freedom

General one dimensional “small” oscillations as the minimum in a potential well. Effective “spring constant.” Motion with damping proportional to velocity, including weak damping, under damping, over damping, and critical damping. Driven oscillations in one dimension, including sinusoidal and general periodic driving functions. Resonance phenomena, including definition and application of the “quality factor” $Q$.

General theory of coupled small oscillations with an arbitrary number of degrees of freedom. Expression as matrix of differential equations. Properties of the $K$ and $M$ matrices. Solution in terms of normal modes, including concepts of “eigenfrequencies” and “eigenmodes.” Motion described as an expansion in normal coordinates.

Dynamics of Rigid Body Rotation

Rigid bodies as a limit of multiparticle systems, based on $\mathbf{v} = \mathbf{\omega} \times \mathbf{r}$. The moment of inertial tensor, principle moments of inertia, and principle axes. Body- and space-fixed coordinates. Euler’s equations in the body-fixed frame and application to free motion and motion with torques. Euler’s angles and motion in the space-fixed frame. General solution of the symmetric top, including precession and nutation.

Nonlinear Systems and Chaos

Surprising behaviors in nonlinear systems, including frequency multiplication, stable and unstable fixed points. The pendulum as a prototype nonlinear oscillator, including damping and forcing. Bifurcation phenomena and bifurcation diagrams. The transition to chaos (and back) demonstrated with the damped driven pendulum. Liapunov exponents and the sensitivity to initial conditions. State space orbits. Chaos as viewed through Poincare sections. Logistics maps as way to characterize nonlinear systems.

Continuum Mechanics of Strings, Fluids, and Elastic Solids

A rod as the infinite-degree-of-freedom limit of springs and masses in one dimension in the Lagrange formalism. The string with small transverse displacement as a Newtonian example. General wave solutions in one dimension, and behavior of a string with fixed ends, analyzed with Fourier “sine” series.

The wave equation in three dimensions. Fluids, material derivatives, and the continuity equation. Inviscid fluids as an example of continuous systems subject only to compressional surface forces, plus body forces. Waves as small pressure disturbances in inviscid fluids. Deformation of elastic solids, and definition of bulk, shear, and Young’s moduli. The stress and strain tensors, and their relationship for linear (small) deformations. Wave phenomena in elastic solids, including longitudinal and transverse cases.

Special Relativity

Principle of the invariant interval. Mathematical framework of special relativity, especially four vectors and the metric tensor. Special four vectors such as four-velocity and four-momentum and their invariant amplitudes. Mass as a relativistic invariant. Massless particles. Applications of the conservation of four-momentum to collision problems.