A string under tension is attached firmly at both ends which do not move. Someone comes along and “plucks” the string by pulling it up in the middle, as shown below, and releasing it from rest:

![String Pluck Diagram]

a) Express the displacement $y(x, t)$ as an expansion in normal modes. Include explicit formulae for the expansion coefficients $A_n$ and $B_n$.

b) You release your finger at time $t = 0$. Plot the shape of the string at the following times:

i) $t = L/(2s)$ (where $s$ is the “speed” of the wave)

ii) $t = L/s$

iii) $t = L/(4s)$

iv) $t = (3/4)(L/s)$

You may need to plot the first several terms of a Fourier expansion to guess the correct shape.

c) Find series expansions for both the kinetic energy $K(t)$ and the potential energy $U(t)$ as a function of time. You can express your answer in terms of any combination of the speed $s$, the tension $T$, the linear mass density $\rho$, and the unstretched string length $L$ that you like.

d) What fraction of the total energy is kinetic or potential at the following times:

i) $t = L/(2s)$

ii) $t = L/s$

iii) $t = L/(4s)$

iv) $t = (3/4)(L/s)$