Homework Assignment Due Thursday, Oct.15

1) Show explicitly that $\nabla (1/r) = -\hat{r}/r^3 = -\hat{r}/r^2$. Class notes from Oct.1 might be useful.

2) Calculate the static electric potential and field far from an electric dipole (Fletcher Fig.1.15):

   a) What is the potential $\phi(\hat{r})$ in terms of the vectors shown in the figure.
   
   b) For $r \gg d$, show that $|\hat{r} \pm (d/2)\hat{k}| \approx r[1 \pm (d/r)(\hat{r} \cdot \hat{k})]^{1/2}$.

   c) Taylor expand for small values of $(d/r)$ to show that the electric potential is

   $\phi(\hat{r}) = \frac{1}{4\pi\epsilon_0} \frac{\hat{p} \cdot \hat{r}}{r^3} = \frac{1}{4\pi\epsilon_0} \frac{\hat{p} \cdot \hat{r}}{r^2} = -\hat{p} \cdot \nabla \left(\frac{1}{r}\right)$

   where $\hat{p} \equiv qd\hat{k}$ is called the electric dipole moment.

   d) Show that $\vec{E} = -\nabla \phi(\hat{r}) = \frac{1}{4\pi\epsilon_0} \frac{3(\hat{p} \cdot \hat{r})\hat{r} - \hat{p}}{r^3}$ in the region $r \gg d$.

   e) Sketch the field vectors $\vec{E}$ in the $xz$ plane (i.e. for $y = 0$).

3) In this problem you will find the properties of an electromagnetic plane wave propagating in an arbitrary direction in free space, where that direction is given by the wave vector $\hat{k} \equiv \hat{k}\hat{k}$.

   a) For a vector field of the form $\vec{F} = \vec{F}_0 f(\hat{k} \cdot \hat{r})$, where $\vec{F}_0$ is a constant vector and $f(u)$ is an arbitrary function of a single variable, show that $\nabla \cdot \vec{F} = 0$ implies that $\hat{k} \cdot \vec{F}_0 = 0$.

   b) For electromagnetic waves with $\vec{E} = \vec{E}_0 \sin (\hat{k} \cdot \hat{r} - \omega t)$ and $\vec{B} = \vec{B}_0 \sin (\hat{k} \cdot \hat{r} - \omega t)$, use Maxwell’s Equations to show that the waves are “transverse”. That is, show that they propagate in a direction that is perpendicular to the direction of the fields $\vec{E}$ and $\vec{B}$.

   c) Use Maxwell’s Equations to show that $\vec{B}_0 = (\hat{k}/c) \times \vec{E}_0$. Explain why this means that $\vec{E}$, $\vec{B}$, and $\hat{k}$ are always mutually perpendicular.