1) These questions have to do with the finite square well which we worked with in class today.

   a) Show that if \( U < \frac{\hbar^2 \pi^2}{8ma^2} \) then there is exactly one energy eigenvalue, i.e. only one bound state solution, and that it has even parity.

   b) Suppose that \( U = \frac{\hbar^2 \pi^2}{ma^2} \). How many bound states are there, and what are their parities?

   You will likely find it easiest to answer this question by making a plot.

   c) Estimate the value of \( ka \) for the ground and first excited states in part (b). You can do this graphically or with whatever numerical program you’d like. (I would use matlab.)

2) One of the most profound problems in Quantum Mechanics has to do with the solution of the simple, one-dimensional harmonic oscillator, where \( V(x) = \frac{1}{2} m \omega^2 x^2 \). (We write \( m \omega^2 \) instead of \( k \) to avoid confusion with the wave number. Here, \( m \) is the mass of the particle as usual, and \( \omega \) just turns out to be the classical angular frequency of the oscillator.) The solution involves the series approach to solving differential equations (see Landshoff, Appendix A) but in this problem we will just look at the ground state.

   a) Show that the wave function \( u(x) = A \exp\left(\frac{-m \omega x^2}{2\hbar}\right) \) is a solution to the time-independent Schrodinger equation, for some value \( A \) and determine the energy eigenvalue. (This is the ground state solution.)

   b) Determine the normalization constant \( A \). You will likely find that Nettel Eq.1.5 is helpful.

   c) Find the position uncertainty \( \Delta x \), the momentum uncertainty \( \Delta p \), and the product \( \Delta x \cdot \Delta p \).

3) Nettel, Exercise 6-3. I will give you a handout that shows how to do part (a), where the Bohr radius and energy levels are derived starting from Bohr’s hypothesis. Part (b) does involve the three dimensional Schrodinger equation, but Nettel gives you the appropriate form for the Laplacian in spherical coordinates. Just take the derivatives carefully.

   \textit{However}, there is a typo in Nettel’s book. In spherical coordinates, the Schrodinger Equation for the hydrogen atom should read

   \[
   \left[ -\frac{\hbar^2}{2mr^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{e^2}{4\pi \varepsilon_0 r}\right]u_1(r) = E_1 u_1(r)
   \]

   That is, the derivative only acts on the wave function, \textit{not} on the potential.