1) In class, we saw that the lowest possible energy for a particle of mass m in one dimensional square well of "radius" a was \( \frac{\hbar^2 \pi^2}{8ma^2} \). Show that

a) for an electron bound in an atom, the energy is close to the "binding energy" (i.e. ionization potential) for a typical atom.

b) for a proton bound in a nucleus, the energy is close to the typical binding energy for a nucleon in a nucleus.

c) it makes no sense for electrons to be bound inside the nucleus.

2) In class we showed that, for the “particle in a box” potential with infinite walls at \( x = \pm a \), the eigenfunctions are \( u_n(x) = A_n \cos\left(\frac{n\pi x}{2a}\right) \) for \( n = 1, 3, \ldots \) and \( u_n(x) = A_n \sin\left(\frac{n\pi x}{2a}\right) \) for \( n = 2, 4, \ldots \).

a) Show by direct integration that these are orthogonal, that is \( \int_{-a}^{a} u_m^* u_n dx = A_n^2 \delta_{mn} \).

b) Show that the normalization constant \( A_n \) for the wave function of the one dimensional square well is \( 1/(\sqrt{\lambda a}) \), independent of the quantum number \( n \), and whether \( n \) is odd or even.

3) Calculate the expectation values of the following for the one dimensional square well:

a) Position \( x \)

b) Momentum \( p = \frac{\hbar}{i} \frac{d}{dx} \)

c) \( x^2 \)

d) \( p^2 \)

4) Calculate the uncertainty in \( x \) and the uncertainty in \( p \) for the one dimensional square well. What is the product of the two uncertainties?