1) A square pulse on a string with fixed ends. A string of length $L$ starts out from rest with a square kink with width $a < L$ in the middle. Set up the problem as follows:

\[ x = 0 \quad \quad x = L/2 \quad \quad x = L \]

a) Determine the Fourier series expansion that gives the initial conditions.

b) Plot the Fourier approximations using different limits of the expansion. Compare them to the square pulse. (By the way, the Heaviside function can be used to draw the square.)

c) Using an appropriate number of Fourier terms, find the expansion for shape of the string as a function of time $t$.

d) Choose a value $a < L/4$ and animate the solution for $t = 0$ to $t = 2(L/s)$ where $s$ is the speed of waves on the string. Briefly explain the motion, including the behavior just after the initial conditions and the behavior at the ends of the string.

2) Fourier Transform of the triangle function. Consider the following function $F(x)$:

\[ \frac{i}{\sqrt{12}} \quad \frac{j}{\sqrt{12}} \quad \frac{\phi}{\sqrt{12}} \]

a) Calculate and plot the Fourier Transform function $A(k)$.

b) Calculate the inverse transform and reproduce (and plot) the function $F(x)$.

c) Calculate the inverse transform with cutoff frequencies $\pi/a$, $2(\pi/a)$, and $8(\pi/a)$.

Compare the result to what you saw for the square pulse and explain the difference.

3) Plotting vector fields. Study the fieldplot and contourplot commands in MAPLE.

a) Make “field” plots of the following vector fields. Which “diverge” and which “curl”?

i) $i + 2j$

ii) $x i$

iii) $x j$

iv) $\frac{x}{(x^2 + y^2)^{1/2}} i + \frac{y}{(x^2 + y^2)^{1/2}} j$

v) $\frac{y}{(x^2 + y^2)^{1/2}} i + \frac{x}{(x^2 + y^2)^{1/2}} j$

b) Find the gradient of the scalar field $\phi(x, y) = x^2 + y^2$. Use the display command to overlay a field plot of the gradient with a contour plot for the scalar field. Briefly explain.