1) **Amplitude and phase of a resonance.**
   a) Define the “width” of a resonance by the frequency spread $\Delta \omega$ between the points below and above the peak, where the amplitude falls to half the peak height. Find an expression for $b \equiv (\Delta \omega)/\omega_0$ in terms of $Q \equiv \omega_0/(2\beta)$ when $Q$ is a large value.
   b) Plot the resonant amplitude and phase, as a function of $\omega/\omega_0$, for forced oscillators with $Q = 10, 50,\text{ and } 200$, and compare the width you see with the formula derived in part (a).

2) **LCR resonant circuit.** This circuit is an electrical analog of the forced mechanical oscillator:

   ![LCR Circuit Diagram]

   a) Write the “equation of motion” for the charge $q$ on the capacitor $C$. The current is driven around the circuit by a potential difference applied by the wave generator $V(t)$.
   b) Determine the parameters $\beta$ and $\omega_0^2$ defined in class (and also used in Greene, Eq.4.12) in terms of $L$, $C$, and $R$. Also derive an expression for the quality factor $Q$.
   c) Find an expression for the rate at which energy is dissipated in the resistor $R$.

3) (Pain, problem 2.6). For an oscillator with a large $Q = \omega_0/(2\beta)$, show that the fractional difference between the resonant frequency and $\omega_0$ is approximately given by $1/(8Q^2)$.

4) **An impulse forcing function.** Perhaps the simplest way to force an oscillator into motion is to “kick” it with a short, sharp blow. Such a force has effectively zero time duration, but still manages to deliver a finite momentum kick to the mass on the oscillator. Mathematically, such an “impulse” force is described in terms of the Dirac Delta function $\delta(t)$. See Greene, Section 4.5.
   a) Set up in MAPLE the differential equation for the forced oscillator, where the forcing function is a series of “kicks” each separated by some time period $\tau$.
   b) Using the initial conditions that the oscillator is at rest and at equilibrium, use `dsolve` to solve this equation, specifying `method=laplace`.
   c) Choose a set of parameters, with $\beta$ and $\omega_0$ corresponding to an underdamped oscillator. Plot the solution over several periods of the (undamped) oscillator $T_0 = (2\pi)/\omega_0$, for the three different values $\tau = 10T_0, T_0$, and $0.1T_0$. Explain the behavior.
   d) For $\tau = T_0$, plot the solution for two values of $\beta$, one much smaller than the other and explain the behavior.