Exercise 8: A Special Field

Due at 5pm on Friday, October 22 in Matt’s mailbox

The flux and divergence of a $\frac{1}{r^2}$ field.

In terms of $x$, $y$, $z$ write down the components $v_x$, $v_y$, $v_z$ of a vector field that points radially away from the origin and has a magnitude $\frac{1}{r^2}$ where $r^2 = x^2 + y^2 + z^2$ is simply the square of the distance from the origin. Show that the divergence of this field is zero everywhere, except (perhaps) at the origin itself. Next, find the flux through a closed spherical surface with radius $R$ centered at the origin, and show that the integral of the flux over this surface is indeed nonzero. Explain why this appears to violate the divergence theorem, unless something is done about the divergence at the origin. Finally, show that if the divergence of the vector field is written as $4\pi \delta(x) \delta(y) \delta(z)$ where $\delta(x)$ is the Dirac delta function, then the divergence theorem is satisfied.

> restart;