You have two hours to complete this exam. There are a total of five problems and you are to solve all of them. *Not all the problems are worth the same number of points.*

You may use *Introductory Astronomy and Astrophysics* (Zeilik & Gregory), *Astronomy: The Evolving Universe* (Zeilik), and class notes and handouts, or other books. You may not share these resources with another student during the test.

*Indicate any figures or tables you use in your calculations. Show all Work!*

GOOD LUCK!

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Total Score: _________ 100
Problem 1 (20 points): Following are 10 brief descriptions of stars. For each one, choose the one spectral type which most closely describes the star. Not all of the spectral types must be used, and you may use some more than once. Indicate your choice with the lower case letter used below:

a: O8V  b: B5I  c: A0V  d: A5I  e: F3I  f: G2V  g: K3V  h: K3III  i: M5I

__h__  1. Spectrum has strong, narrow calcium lines
__f__  2. The Sun
__c__  3. Continuum part of the spectrum peaks at 290 nm
__i__  4. Coolest surface temperature in the list
__i__  5. Largest radius star in the list
__c__  6. Strongest hydrogen lines in the list
__a__  7. A very massive star with weak hydrogen lines
__g__  8. A star with a smaller radius than the Sun
__e__  9. Surface temperature close to 7000 K
__b__  10. Hydrogen and helium lines with about equal strength
Problem 2 (15 points): Calculate the effective surface temperature of a star with radius \(1.39 \times 10^9\) m and which radiates \(1.56 \times 10^{27}\) Joules per second, integrated over all wavelengths.

This could be done directly from the Stefan-Boltzmann law ...

\[
L = 4\pi R^2 \sigma T^4 = 4\pi (1.39 \times 10^9)^2 \sigma T^4 = 1.56 \times 10^{27}
\]

... which leads to \(T \approx 5800\) K with everything in SI units.

A different way is to write it all in terms of solar units, that is ...

\[
\frac{L}{L_{\text{SUN}}} = \frac{4\pi R^2 \sigma T^4}{4\pi R_{\text{SUN}}^2 \sigma T_{\text{SUN}}^4} = \left(\frac{R}{R_{\text{SUN}}}\right)^2 \left(\frac{T}{T_{\text{SUN}}}\right)^4
\]

... which reduces to

\[
4 = (2)^2 \left(\frac{T}{T_{\text{SUN}}}\right)^4
\]

or just \(T = T_{\text{SUN}} \approx 5800\) K
**Problem 3 (20 points):** An A0 main sequence star has an apparent visual magnitude of 12.5 and an apparent blue magnitude of 13.3. The spectral class is assigned using the absorption lines.

**a (2 points)** What is the value of the observed color index $B-V$?

$$B-V = 13.3 - 12.5 = 0.8$$

**b (3 points)** What is the value of the intrinsic color index $B-V$?

$B-V = 0$ for an A0 main sequence star; See also Table A4-3 in Zeilik.

**c (3 points)** What is the value of the star’s absolute visual magnitude?

$M_V = +0.6$ from Table A4-3, or $M_V \approx +1$ from an HR diagram in absolute magnitude, like Fig. 13-9C, 13-11B, or the left axis on 13-14B.

**d (4 points)** Estimate the distance to the star if there were no interstellar reddening.

$$12.5 - M_V = 5 \log d - 5 \quad \text{leads to } d=2.4 \text{ kpc (}M_V = +0.6) \text{ or } d=2.0 \text{ kpc (}M_V = +1)$$

**e (8 points)** Estimate the distance to the star including interstellar reddening, based on the data given in this problem.

With interstellar reddening, we modify the distance formula to be

$$12.5 - M_V = 5 \log d - 5 + A_V$$

where we can determine $A_V$ from the amount of reddening, that is

$$A_V = 3 (0.8 - 0) = 2.4$$

which leads to $d = 738$ pc (for $M_V = +0.6$).
**Problem 4 (25 points):** Two stars form an eclipsing binary pair, viewed edge on. Star #1 has a surface temperature of 15,000 K and star #2 has a surface temperature of 5000 K. The cooler star is a giant with a radius four times as large as the hotter star.

**a (5 points)** Is the primary (deeper) intensity minimum found when star #2 eclipses star #1, or the other way around? Explain your reasoning.

When both stars are visible, the luminosity is \( L = L_1 + L_2 \) where

\[
\frac{L_1}{L_2} = \left( \frac{R_1}{R_2} \right)^2 \left( \frac{T_1}{T_2} \right)^4 = \frac{81}{16}
\]

that is, star #1 is much brighter than star #2. The deepest minimum in intensity therefore happens when star #1 is behind star #2.

**b (5 points)** Find the ratio of the luminosity \( L_P \) at the primary minimum to the maximum luminosity \( L \).

The luminosity at the primary minimum is just \( L_2 \) since star #1 is hidden. Therefore

\[
\frac{L_P}{L} = \frac{L_2}{L_1 + L_2} = \frac{1}{81/16 + 1} = \frac{16}{97} = 0.165
\]

**c (10 points)** Find the ratio of the luminosity \( L_S \) at the secondary minimum to the maximum luminosity \( L \).

This is trickier since not all of star #2 is hidden at the secondary minimum because it has a larger radius. The fraction of the surface area that is hidden is just \( (R_1/R_2)^2 = 1/16 \) so

\[
\frac{L_S}{L} = \frac{L_1 + \frac{15}{16} L_2}{L_1 + L_2} = \frac{81/16 + 15/16}{81/16 + 1} = \frac{96}{97} = 0.9897
\]

**d (5 points)** What is the change in magnitudes at the primary minimum?

Since both stars in the binary system are at the same distance, the difference in magnitudes is just derived from the log of the ratio of the luminosities. That is

\[
\Delta m = 2.5 \log \frac{L}{L_P} = 1.96
\]

a (15 points) Use the figure to estimate the distance to the globular cluster. You can ignore the effects of interstellar reddening.

This is just like calculating the distance to the Pleiades, which we did in studio, but in globular clusters, only the low mass stub of the main sequence is visible. You can use any point on that stub to get the distance. For example, at B-V=0.5, the apparent visual magnitude $V \approx 18$. The absolute visual magnitude for a main sequence star with B-V=0.5 can be gotten from, for example, Fig.13-15 (for which my eye says $M_V \approx +5$) or by interpolating from Table A4-3 (for which B-V=0.5 seems to correspond to around F7 on the main sequence, so $M_V \approx +4$). Therefore

$$18 - M_V = 5 \log d - 5$$

implies $d = 3.98$ kpc (for $M_V \approx +5$) or $d = 6.31$ kpc (for $M_V \approx +4$).

b (5 points) Is this a reasonable value for the distance? Explain your reasoning.

This is a reasonable value (which is good since the data was taken from a real paper in the Astrophysical Journal). Globular clusters hang out surrounding the center of our galaxy, outside the galactic plane. We are around 8.5 kpc from the galactic center, so it makes sense that a globular cluster should be several kpc away from us.