Please state clearly all assumptions made in order for full credit to be given.

<table>
<thead>
<tr>
<th>Problem</th>
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<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>Total</td>
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Problem #1 (25)

Consider the three points $P_1 (1, -4, 7)$, $P_2 (2, -1, 3)$, and $P_3 (-2, -6, 4)$ in a 3D Cartesian coordinate system,

(a) Determine vector $\vec{A}$ with initial point $P_1$ and terminal point $P_2$;  
(b) Determine vector $\vec{B} = P_1 - P_3/2$;  
(c) Determine the angle, $\theta$, between vectors $\vec{A}$ and $\vec{B}$. Provide your result in degrees;  
(d) Determine the vector component of $\vec{B}$ along $\vec{A}$;  
(e) Determine the vector component of $\vec{B}$ orthogonal to $\vec{A}$.

Solution:

(a) $\vec{A} = P_2 - P_1$

$$ = (2, -1, 3) - (1, -4, 7) = (1, 3, -4) .$$

(b) $\vec{B} = P_1 - P_3/2 = (1, -4, 7) - (-2, -6, 4)/2 = (1, -4, 7) - (-1, -3, 2) = (2, -1, 5).$

(c) $\cos\theta = \vec{A} \cdot \vec{B} / AB$

$$\vec{A} \cdot \vec{B} = 1 \cdot 2 + 3 \cdot (-1) + (-4) \cdot 5 = 2 - 3 - 20 = -21$$

$$A = \sqrt{1^2 + 3^2 + (-4)^2} = 5.099$$

$$B = \sqrt{2^2 + (-1)^2 + 5^2} = 5.477$$

$$\cos\theta = \frac{-21}{5.099 \cdot 5.477} = -0.752 \Rightarrow \theta = 138.8^\circ$$

(d) $\vec{B}_{A} = (\vec{B} \cdot \vec{u}_A) \vec{u}_A = \frac{\vec{B} \cdot \vec{A}}{A^2} \vec{A}$

$$= \frac{-21}{26} (1, 3, -4) = (-0.808, -2.423, 3.231)$$

(e) $\vec{B}_{\perp A} = \vec{B} - \vec{B}_{A}$

$$= (2, -1, 5) - (-0.808, -2.423, 3.231) = (2.808, 1.423, 1.769)$$
Problem #2 (25)

Determine the magnitude and coordinate direction angles of the resultant force acting on the pipe assembly.

Solution:

\[ F_{1x} = F_1 \left(\frac{4}{5}\right) = 480 \text{ lb} \]  
(2pts)

\[ F_{1y} = 0 \text{ lb} \]  
(2pts)

\[ F_{1z} = F_1 \left(\frac{3}{5}\right) = 360 \text{ lb} \]  
(2pts)

\[ F_{2x} = F_2 \cos \alpha_2 = 400 \cos 60^\circ = 200 \text{ lb} \]  
(2pts)

\[ F_{2z} = F_2 \cos \gamma_2 = 400 \cos 120^\circ = -200 \text{ lb} \]  
(2pts)

\[ \cos^2 \alpha_2 + \cos^2 \beta_2 + \cos^2 \gamma_2 = 1 \Rightarrow \cos^2 \beta_2 = \frac{1}{2} \]

From figure, \( 0^\circ < \beta_2 < 90^\circ \). Hence,

\[ \cos \beta_2 = \frac{\sqrt{2}}{2} \]

\[ F_{2y} = F_2 \cos \beta_2 = 400 \left(\frac{\sqrt{2}}{2}\right) = 282.8 \text{ lb} \]

(5pts)

\[ \vec{F}_1 = \left(480\hat{i} + 0\hat{j} + 360\hat{k}\right) \text{ lb}, \quad \vec{F}_2 = \left(200\hat{i} + 282.8\hat{j} - 200\hat{k}\right) \text{ lb} \]

\[ \therefore \vec{F}_R = \left(680\hat{i} + 282.8\hat{j} + 160\hat{k}\right) \text{ lb} \]  
(2pts)

\[ F_R = \sqrt{(680)^2 + (282.8)^2 + (160)^2} = 754 \text{ lb} \]  
(2pts)

\[ \cos \alpha = 680/754 = 0.902 \Rightarrow \alpha = 25.6^\circ \]  
(2pts)

\[ \cos \beta = 282.8/754 = 0.375 \Rightarrow \beta = 68.0^\circ \]  
(2pts)

\[ \cos \gamma = 160/754 = 0.212 \Rightarrow \gamma = 77.7^\circ \]  
(2pts)
Problem #3 (25)

Two street lights are attached by cables to two poles as shown. The light at $B$ has a weight of 40 lb and the light at $C$ has a weight of 60 lb. The cable $BC$ has to be fully horizontal. Determine the forces within the cables $AB$, $BC$ and $CD$. (21 points)

Determine the height $h$ of the post of $D$ needed to ensure that $BC$ is fully horizontal. (4 points)

Draw all needed free-body-diagrams separately from the figure above.

$\theta = \arctan(3/6) = 26.5651^\circ$  \hspace{1cm}  $\sin(\theta) = 0.4472$  \hspace{1cm}  $\cos(\theta) = 0.8949$

FBD of B

$\Sigma F_y = 0$

$T_{BA} \sin(\theta) - 40 \text{ lb} = 0$

$T_{BA} = 89.4427 \text{ lb}$

$\Sigma F_x = 0$

$-T_{BA} \cos(\theta) + T_{BC} = 0$

$T_{BC} = 80.0000 \text{ lb}$
FBD of C

$\sum F_x = 0$

$-T_{BC} + T_{CD} \cos(\phi) = 0$

$T_{CD} \cos(\phi) = 80$  \hspace{1cm} \text{(Eq. 1)}

$\sum F_y = 0$

$T_{CD} \sin(\phi) - 60 \text{ lb} = 0$

$T_{CD} \sin(\phi) = 60 \text{ lb}$  \hspace{1cm} \text{(Eq. 2)}

Eq.2/Eq1 $\rightarrow \tan(\phi) = 60/80$  \hspace{1cm} \phi = 36.8699^\circ

$T_{CD} = 100 \text{ lb}$

$\tan(\phi) = (h-17)/8 = 60/80$

$h = 23 \text{ ft}$
Problem #4 (25)

Solve the following system of linear equations using the Gauss-Jordan Elimination method:
3 \( x \) + 3 \( y \) + 2/3 \( z \) = 3
\( x \) + 1/2 \( y \) + 1 \( z \) = 2
5 \( x \) + 2 \( y \) + 1 \( z \) = 1/4

a) Write the augmented matrix (5 points)
b) Use elementary row operations to obtain the reduced row echelon form (17 points)
c) Write the solution for the three variables \( x \), \( y \), and \( z \) (3 points)

Show all details of the elementary row operations (no credit will be given for unsupported answers).

Augmented Matrix

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<tr>
<th>3</th>
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<th>3</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1/2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1</td>
<td>1/4</td>
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Elementary row operations

Divide row1 by 3

<table>
<thead>
<tr>
<th>3</th>
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<th>3</th>
<th>R1 ( \leftarrow ) R1/3</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1/2</td>
<td>1</td>
<td>2</td>
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</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1</td>
<td>1/4</td>
<td></td>
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</table>

Add \((-1 \times \text{row1})\) to row2

<table>
<thead>
<tr>
<th>1</th>
<th>1</th>
<th>2/9</th>
<th>1</th>
<th>R2 ( \leftarrow ) R2-1*R1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/2</td>
<td>1</td>
<td>2</td>
<td>R2 ( \leftarrow ) R2-1*R1</td>
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<tr>
<td>5</td>
<td>2</td>
<td>1</td>
<td>1/4</td>
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Add \((-5 \times \text{row1})\) to row3

<table>
<thead>
<tr>
<th>1</th>
<th>1</th>
<th>2/9</th>
<th>1</th>
<th>R3 ( \leftarrow ) R3-5*R1</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>-1/2</td>
<td>7/9</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1</td>
<td>1/4</td>
<td>R3 ( \leftarrow ) R3-5*R1</td>
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</table>

Divide row2 by \(-1/2\)

<table>
<thead>
<tr>
<th>1</th>
<th>1</th>
<th>2/9</th>
<th>1</th>
<th>R2 ( \leftarrow ) R2/(-1/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1/2</td>
<td>7/9</td>
<td>1</td>
<td>R2 ( \leftarrow ) R2/(-1/2)</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
<td>-19/4</td>
<td>1</td>
<td></td>
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</table>

Add \((3 \times \text{row2})\) to row3

<table>
<thead>
<tr>
<th>1</th>
<th>1</th>
<th>2/9</th>
<th>1</th>
</tr>
</thead>
</table>
Divide row3 by $-\frac{43}{9}$

Add $(\frac{14}{9} \times \text{row3})$ to row2

Add $(-\frac{2}{9} \times \text{row3})$ to row1

Add $(-1 \times \text{row2})$ to row1

RREF

Solution for the three variables $x$, $y$, and $z$

$x = -1$, $y = \frac{3}{2}$, $z = \frac{9}{4}$