ENGR-1100 Introduction to Engineering Analysis

Lecture 26
CHARACTERISTICS OF DRY FRICTION & PROBLEMS INVOLVING DRY FRICTION

Today’s Objective:
Students will be able to:
a) Understand the characteristics of dry friction
b) Draw a FBD including friction.
c) Solve problems involving friction.

In-Class Activities:
• Reading Quiz
• Applications
• Characteristics of Dry Friction
• Problems involving Dry Friction
• Concept Quiz
• Group Problem Solving
• Attention Quiz
In designing a brake system for a bicycle, car, or any other vehicle, it is important to understand the frictional forces involved.

For an applied force on the bike tire brake pads, how can we determine the magnitude and direction of the resulting friction force?
The rope is used to tow the refrigerator.

In order to move the refrigerator, is it best to pull up as shown, pull horizontally, or pull downwards on the rope?

What physical factors affect the answer to this question?
Friction is defined as a force of resistance acting on a body which prevents or resists the slipping of a body relative to a second body.

Experiments show that frictional forces act tangent (parallel) to the contacting surface in a direction opposing the relative motion or tendency for motion.

For the body shown in the figure to be in equilibrium, the following must be true: \( F = P \), \( N = W \), and \( W \times x = P \times h \).
To study the characteristics of the friction force $F$, let us assume that tipping does not occur (i.e., “$h$” is small or “$a$” is large).

Then we gradually increase the magnitude of the force $P$.

Typically, experiments show that the friction force $F$ varies with $P$, as shown in the right figure above.
CHARACTERISTICS OF DRY FRICTION
(continued)

The **maximum friction force** is attained just before the block begins to move (a situation that is called “impending motion”). The value of the force is found using $F_s = \mu_s N$, where $\mu_s$ is called the coefficient of static friction. The value of $\mu_s$ depends on the two materials in contact.

Once the block begins to move, the frictional force typically drops and is given by $F_k = \mu_k N$. The value of $\mu_k$ (coefficient of kinetic friction) is less than $\mu_s$. 
It is also very important to note that the friction force may be less than the maximum friction force. So, just because the object is not moving, don’t assume the friction force is at its maximum of $F_s = \mu_s \ N$ unless you are told or know motion is impending!
DETERMING $\mu_s$ EXPERIMENTALLY

If the block just begins to slip, the maximum friction force is $F_s = \mu_s N$, where $\mu_s$ is the coefficient of static friction.

Thus, when the block is on the verge of sliding, the normal force $N$ and frictional force $F_s$ combine to create a resultant $R_s$.

From the figure,

$$\tan \phi_s = \left( \frac{F_s}{N} \right) = \left( \frac{\mu_s N}{N} \right) = \mu_s$$
DETERMINING $\mu_s$ EXPERIMENTALLY (continued)

A block with weight $w$ is placed on an inclined plane. The plane is slowly tilted until the block just begins to slip.

The inclination, $\theta_s$, is noted. Analysis of the block just before it begins to move gives (using $F_s = \mu_s N$):

\[ + \sum F_y = N - W \cos \theta_s = 0 \]
\[ + \sum F_x = \mu_s N - W \sin \theta_s = 0 \]

Using these two equations, we get

$\mu_s = (W \sin \theta_s) / (W \cos \theta_s) = \tan \theta_s$

This simple experiment allows us to find the $\mu_s$ between two materials in contact.
Steps for solving equilibrium problems involving dry friction:

1. Draw the necessary free body diagrams. Make sure that you show the friction force in the correct direction (it always opposes the motion or impending motion).

2. Determine the number of unknowns. Do not assume that $F = \mu_s N$ unless the impending motion condition is given.

3. Apply the equations of equilibrium and appropriate frictional equations to solve for the unknowns.
IMPENDING TIPPING versus SLIPPING

For a given $W$ and $h$ of the box, how can we determine if the block will slide or tip first? In this case, we have four unknowns ($F$, $N$, $x$, and $P$) and only three E-of-E.

Hence, we have to make an assumption to give us another equation (the friction equation!). Then we can solve for the unknowns using the three E-of-E. Finally, we need to check if our assumption was correct.
Assume: **Slipping** occurs

**Known:** \( F = \mu_s N \)

**Solve:** \( x, P, \text{ and } N \)

**Check:** \( 0 \leq x \leq b/2 \)

Or

Assume: **Tipping** occurs

**Known:** \( x = b/2 \)

**Solve:** \( P, N, \text{ and } F \)

**Check:** \( F \leq \mu_s N \)
EXAMPLE

**Given:** Crate weight = 250 lb, 
\( \mu_s = 0.4 \)

**Find:** The maximum force \( P \) that can be applied without causing movement of the crate.

**Plan:** ??

a) Draw a FBD of the box.
b) Determine the unknowns.
c) Make your friction assumptions.
d) Apply E-of-E (and friction equations, if appropriate ) to solve for the unknowns.
e) Check assumptions, if required.
There are four unknowns: P, N, F and x.

First, let’s assume the crate slips. Then the friction equation is \( F = \mu_s N = 0.4 \, N \).
EXAMPLE (continued)

\[ + \sum F_x = P - 0.4N = 0 \]
\[ + \sum F_y = N - 250 = 0 \]

Solving these two equations gives:

\[ P = 100 \text{ lb} \quad \text{and} \quad N = 250 \text{ lb} \]

\[ \sum M_o = -100 (4.5) + 250 (x) = 0 \]

Check: \( x = 1.8 \geq 1.5 \): No slipping will occur since \( x > 1.5 \)
EXAMPLE (continued)

Since tipping occurs, here is the correct FBD:

\[ + \rightarrow \sum F_x = P - F = 0 \]
\[ + \uparrow \sum F_y = N - 250 = 0 \]

These two equations give:
\[ P = F \quad \text{and} \quad N = 250 \text{ lb} \]

\[ + \sum M_O = -P (4.5) + 250 (1.5) = 0 \]

\[ P = 83.3 \text{ lb}, \quad \text{and} \quad F = 83.3 \text{ lb} < \mu_s N = 100 \text{ lb} \]
1. A friction force always acts _____ to the contact surface.
   A) Normal  B) At 45°
   C) Parallel  D) At the angle of static friction

2. If a block is stationary, then the friction force acting on it is ________.
   A) \( \leq \mu_s N \)  B) \( = \mu_s N \)
   C) \( \geq \mu_s N \)  D) \( = \mu_k N \)
CONCEPT QUIZ

1. A 100 lb box with a wide base is pulled by a force $P$ and $\mu_s = 0.4$. Which force orientation requires the least force to begin sliding?

   A) $P(A)$  
   B) $P(B)$  
   C) $P(C)$  
   D) Can not be determined

2. A ladder is positioned as shown. Please indicate the direction of the friction force on the ladder at B.

   A) $\uparrow$  
   B) $\downarrow$  
   C) $\rightarrow$  
   D) $\leftarrow$
1. A 10 lb block is in equilibrium. What is the magnitude of the friction force between this block and the surface?
   A) 0 lb  B) 1 lb  C) 2 lb  D) 3 lb

2. The ladder AB is positioned as shown. What is the direction of the friction force on the ladder at B.
   A)  B)  C) ←  D) ↑
GROUP PROBLEM SOLVING

Given: Dresser weight = 90 lb, man’s weight = 150 lb. \( \mu_s = 0.25 \).

Find: The smallest magnitude of \( F \) needed to move the dresser if \( \theta = 30^\circ \).

Also determine the smallest coefficient of static friction between his shoes and the floor so that he does not slip.

Plan: a) Draw FBDs of the dresser and man.
b) Determine the unknowns.
d) Apply E-of-E to solve for the unknowns.
GROUP PROBLEM SOLVING (continued)

FBD of the man

\[ \sum F_x = F \cos 30 - 0.25 N = 0 \]
\[ + \uparrow \sum F_y = N - 90 - F \sin 30 = 0 \]

These two equations give:

\[ F = 30.36 \text{ lb} = 30.4 \text{ lb} \]
\[ N = 105.1 \text{ lb} \]

FBD of the dresser

Dresser : 

\[ \sum F_x = F \cos 30 - 0.25 N = 0 \]
\[ + \uparrow \sum F_y = N - 90 - F \sin 30 = 0 \]
GROUP PROBLEM SOLVING (continued)

FBD of the man

\[ \begin{align*}
\text{Man:} & \quad + \rightarrow \sum F_x = \mu_m N_m - 30.363 \cos(30) = 0 \\
& \quad + \uparrow \sum F_y = N_m - 150 + 30.363 \sin(30) = 0
\end{align*} \]

These two equations give:

\[ N_m = 134.8 \text{ lb} \]
\[ \mu_m = 0.195 \]