ENGR-1100 Introduction to Engineering Analysis

Lecture 19
SIMPLE TRUSSES, THE METHOD OF JOINTS, & ZERO-FORCE MEMBERS

Today’s Objectives:

- Students will be able to:
  - Define a simple truss.
  - Determine the forces in members of a simple truss.
  - Identify zero-force members.

In-Class Activities:

- Reading Quiz
- Applications
- Simple Trusses
- Method of Joints
- Zero-force Members
- Concept Quiz
- Group Problem Solving
- Attention Quiz
APPLICATIONS

Trusses are commonly used to support roofs.

For a given truss geometry and load, how can you determine the forces in the truss members and thus be able to select their sizes?

A more challenging question is, that for a given load, how can we design the trusses’ geometry to minimize cost?
APPLICATIONS (continued)

Trusses are also used in a variety of structures like cranes and the frames of aircraft or the space station.

How can you design a light weight structure satisfying load, safety, cost specifications, is simple to manufacture, and allows easy inspection over its lifetime?
A truss is a structure composed of slender members joined together at their end points. If a truss, along with the imposed load, lies in a single plane (as shown at the top right), then it is called a planar truss. A simple truss is a planar truss which begins with a triangular element and can be expanded by adding two members and a joint. For these trusses, the number of members (M) and the number of joints (J) are related by the equation $M = 2J - 3$. 
ANALYSIS & DESIGN ASSUMPTIONS

When designing the members and joints of a truss, first it is necessary to determine the forces in each truss member. This is called the force analysis of a truss. When doing this, two assumptions are made:

1. All loads are applied at the joints. The weight of the truss members is often neglected as the weight is usually small as compared to the forces supported by the members.

2. The members are joined together by smooth pins. This assumption is satisfied in most practical cases where the joints are formed by bolting the ends together.

With these two assumptions, the members act as two-force members. They are loaded in either tension or compression. Often compressive members are made thicker to prevent buckling.
When using the method of joints to solve for the forces in truss members, the equilibrium of a joint (pin) is considered. All forces acting at the joint are shown in a FBD. This includes all external forces (including support reactions) as well as the forces acting in the members. Equations of equilibrium ($\sum F_x = 0$ and $\sum F_y = 0$) are used to solve for the unknown forces acting at the joints.
STEPS FOR ANALYSIS

1. If the truss’s support reactions are not given, draw a FBD of the entire truss and determine the support reactions (typically using scalar equations of equilibrium).

2. Draw the free-body diagram of a joint with one or two unknowns. Assume that all unknown member forces act in tension (pulling on the pin) unless you can determine by inspection that the forces are compression loads.

3. Apply the scalar equations of equilibrium, \( \sum F_x = 0 \) and \( \sum F_y = 0 \), to determine the unknown(s). If the answer is positive, then the assumed direction (tension) is correct, otherwise it is in the opposite direction (compression).

4. Repeat steps 2 and 3 at each joint in succession until all the required forces are determined.
ZERO-FORCE MEMBERS
(Section 6.3)

If a joint has only two non-collinear members and there is no external load or support reaction at that joint, then those two members are zero-force members. In this example members DE, DC, AF, and AB are zero force members.

You can easily prove these results by applying the equations of equilibrium to joints D and A.

Zero-force members can be removed (as shown in the figure) when analyzing the truss.
If three members form a truss joint for which two of the members are collinear and there is no external load or reaction at that joint, then the third non-collinear member is a zero force member, e.g., DA.

Again, this can easily be proven. One can also remove the zero-force member, as shown, on the left, for analyzing the truss further.

Please note that zero-force members are used to increase stability and rigidity of the truss, and to provide support for various different loading conditions.
EXAMPLE

**Given:** Loads as shown on the truss

**Find:** The forces in each member of the truss.

**Plan:**

1. Check if there are any zero-force members.
2. First analyze pin D and then pin A
3. Note that member BD is zero-force member. $F_{BD} = 0$
4. Why, for this problem, do you not have to find the external reactions before solving the problem?
EXAMPLE (continued)

\[ + \rightarrow \sum F_X = -450 + F_{CD} \cos 45^\circ - F_{AD} \cos 45^\circ = 0 \]

\[ + \uparrow \sum F_Y = -F_{CD} \sin 45^\circ - F_{AD} \sin 45^\circ = 0 \]

\[ F_{CD} = 318 \text{ lb (Tension) or (T)} \]

and \[ F_{AD} = -318 \text{ lb (Compression) or (C)} \]
EXAMPLE (continued)

Analyzing pin A:

\[ \sum F_x = F_{AB} + (-318) \cos 45^\circ = 0; \quad F_{AB} = 225 \text{ lb (T)} \]

Could you have analyzed Joint C instead of A?
READING QUIZ

1. One of the assumptions used when analyzing a simple truss is that the members are joined together by __________.
   A) Welding       B) Bolting       C) Riveting
   D) Smooth pins   E) Super glue

2. When using the method of joints, typically __________ equations of equilibrium are applied at every joint.
   A) Two          B) Three
   C) Four         D) Six
1. Truss ABC is changed by decreasing its height from $H$ to $0.9\ H$. Width $W$ and load $P$ are kept the same. Which one of the following statements is true for the revised truss as compared to the original truss?

A) Force in all its members have decreased.

B) **Force in all its members have increased.**

C) Force in all its members have remained the same.

D) None of the above.
2. For this truss, determine the number of zero-force members.

A) 0  B) 1  C) 2
D) 3  E) 4
ATTENTION QUIZ

1. Using this FBD, you find that $F_{BC} = -500 \text{ N}$. Member BC must be in __________.
   
   A) Tension
   
   B) Compression
   
   C) Cannot be determined

2. When supporting the same magnitude of force, truss members in compression are generally made _______ as compared to members in tension.
   
   A) Thicker
   
   B) Thinner
   
   C) The same size
GROUP PROBLEM SOLVING

Given: Loads as shown on the truss

Find: Determine the force in all the truss members (do not forget to mention whether they are in T or C).

Plan:

a) Check if there are any zero-force members.
   Is Member CE zero-force member?

b) Draw FBDs of pins D, C, and E, and then apply E-of-E at those pins to solve for the unknowns.
Analyzing pin D:

\[ \sum F_x = F_{DE} \left( \frac{3}{5} \right) - 600 = 0 \]

\[ F_{CD} = 1000 \text{ N} = 1.00 \text{ kN (C)} \]

\[ \sum F_y = 1000 \left( \frac{4}{5} \right) - F_{CD} = 0 \]

\[ F_{DE} = 800 \text{ N} = 0.8 \text{ kN (T)} \]
GROUP PROBLEM SOLVING (continued)

Analyzing pin C:

\[ \sum F_X = F_{CE} - 900 = 0 \]

\[ F_{CE} = 900 \text{ N} = 0.90 \text{ kN (C)} \]

\[ \sum F_Y = 800 - F_{BC} = 0 \]

\[ F_{BC} = 800 \text{ N} = 0.80 \text{ kN (T)} \]
GROUP PROBLEM SOLVING (continued)

Analyzing pin E:

\[ \sum F_X = F_{AE} \left(\frac{3}{5}\right) + F_{BE} \left(\frac{3}{5}\right) - 1000 \left(\frac{3}{5}\right) - 900 = 0 \]

\[ \sum F_Y = F_{AE} \left(\frac{4}{5}\right) - F_{BE} \left(\frac{4}{5}\right) - 1000 \left(\frac{4}{5}\right) = 0 \]

Solving these two equations, we get

\( F_{AE} = 1750 \text{ N} = 1.75 \text{ kN (C)} \)

\( F_{BE} = 750 \text{ N} = 0.75 \text{ kN (T)} \)