ENGR-1100 Introduction to Engineering Analysis

Lecture 27

Notes courtesy of: Prof. Yoav Peles
Today Lecture outline

• Belt friction
  • Flat belts
  • V-belts
Belt friction

Fig. P9-89
\[ \sum F_r = 0 \quad \Rightarrow \quad \Delta P - T \sin \left( \frac{\Delta \theta}{2} \right) - (T + \Delta T) \sin \left( \frac{\Delta \theta}{2} \right) = 0 \]

\[ \Delta P = 2T \sin \left( \frac{\Delta \theta}{2} \right) + \Delta T \sin \left( \frac{\Delta \theta}{2} \right) \] (a)
\[ \sum F_\theta = 0 \]
\[ (T + \Delta T) \cos \left( \frac{\Delta \theta}{2} \right) - T \cos \left( \frac{\Delta \theta}{2} \right) - \Delta F = 0 \]
\[ \Delta T \cos \left( \frac{\Delta \theta}{2} \right) = \Delta F \quad \text{(b)} \]
Assuming slip is impending

$$\Delta F = \mu_s \Delta P$$

$$\Delta P = 2T \sin \left( \frac{\Delta \theta}{2} \right) + \Delta T \sin \left( \frac{\Delta \theta}{2} \right) \quad \text{(a)}$$

$$\Delta T \cos \left( \frac{\Delta \theta}{2} \right) = \Delta F \quad \text{(b)}$$

Combining equation (a) and (b) yields:

$$\Delta T \cos(\Delta \theta) = \mu_s 2T \sin(\Delta \theta) + \mu_s \Delta \sin(\Delta \theta)$$
Combining equation (a) and (b) yields:

$$\Delta T \cos(\Delta \theta) = \mu_s T \sin(\Delta \theta) + \mu_s \Delta T \sin(\Delta \theta)$$

Dividing by $\Delta \theta$:

$$\left(\frac{\Delta T}{\Delta \theta}\right) \cos \left(\frac{\Delta \theta}{2}\right) = \mu_s T \left(\frac{\sin \left(\frac{\Delta \theta}{2}\right)}{\Delta \theta/2}\right) + \mu_s \Delta T \left(\frac{\sin \left(\frac{\Delta \theta}{2}\right)}{\Delta \theta/2}\right)$$

$$\lim_{\Delta \theta \to 0} \frac{\Delta T}{\Delta \theta} \to \frac{dt}{d\theta} \quad \lim_{\chi \to 0} \cos \chi \to 1 \quad \lim_{\chi \to 0} \sin \chi \to 1$$

$$dT/d\theta = \mu_s T + O(\Delta T) \quad \Delta T \to 0 \quad dT/d\theta = \mu_s T$$

Rearranging:

$$dT/T = \mu_s d\theta$$
\[ \frac{dT}{T} = \mu_s \, d\theta \]

Integrating

\[ \ln \left( \frac{T_2}{T_1} \right) = \mu_s (\theta_2 - \theta_1) = \mu_s \beta \]

Where: \( \beta = \theta_2 - \theta_1 \)

Or

\[ T_2 = T_1 e^{\mu_s \beta} \]  

\( (c) \)
V-belt

\[ T_2 = T_1 e^{(\mu_s)_{\text{enh}} \beta} \]

Where \((\mu_s)_{\text{enh}} = \left[ \frac{s}{\sin(\alpha/2)} \right]\) is the enhanced coefficient of friction.
Example

A rope attached to a 500-lb block passes over a frictionless pulley and is wrapped for one full turn around a fixed post as shown. If the coefficient of friction between the rope and the post is 0.25, determine

a) The minimum force $P$ that must be used to keep the block from falling. b) The minimum force $P$ that must be used to begin to raise the block.
Solution

A free-body diagram for the block:

\[ \uparrow \sum F_y = T - W = 0 \]

\[ T = 500 \text{ lb} \]

From equation (c):

\[ T_2 = T_1 e^{\mu \beta} = T_1 e^{0.25 (2\pi)} = 4.8104 \ T_1 \]

(a) During lowering of the block:

\[ P_{\text{min}} = T_1 = \frac{500}{4.8104} \approx 103.9 \text{ lb} \]
(b) During raising the block:

\[ P_{\text{min}} = T_2 = 500 \times 4.8104 \approx 2,405 \text{ lb} \]
Example:

The band brake is used to control the rotation of a drum. The coefficient of friction between the belt and the drum is 0.35 and the weight of the handle is 15 N. If a force of 200 N is applied to the end of the handle, determine the maximum torque for which no motion occurs if the torque is applied

(a) Clockwise

(b) Counterclockwise
Solution

\[ \beta = \frac{180 + 30}{360} (2\pi) = 3.665 \text{ rad} \]

\[ \mu \beta = 0.35 (3.665) = 1.2828 \]

From equation (c):

\[ T_2 = T_1 e^{\mu \beta} = T_1 e^{1.2828} = 3.606 T_1 \]

(a) From a free-body diagram of the handle

\[ \Sigma M_A = 200(675) - 15(225) - T(75) = 0 \]

\[ T = 1755.0 \text{ N} \]
(a) From a free-body diagram of the drum when the rotation of the drum is clockwise:

\[ T_2 = T \]

\[ T_1 = \frac{T_2}{3.607} = \frac{1755.0}{3.6067} = 486.6 \text{ N} \]

\[ T_{\text{max}} = (T_2 - T_1)R = (1755.0 - 486.6)(0.150) = 190.3 \text{ N \cdot m} \]
(b) From a free-body diagram of the drum when the rotation of the drum is counterclockwise:

\[ T_1 = T \]

\[ T_2 = 3.6067 \cdot T_1 = 3.6067 \cdot 1755.0 = 6329.8 \text{ N} \]

\[ T_{\text{max}} = (T_2 - T_1)R = (6329.8 - 1755.0) \cdot 0.150 = 686 \text{ N} \cdot \text{m} \]