Today Lecture outline

• Composite areas
Distributed area: the moment about a point

\[ dM_i = x_i dA_i \]

\[ M_y = \sum_{i=1}^{n} x_i dA_i \]

\[ M_y = \int_{A} x_i dA_i \]
Composite areas

\[ A = \int_{x=0}^{x=b} dA = \int_{x=0}^{x=a} dA + \int_{x=a}^{x=b} dA = A_{\text{tri}} + A_{\text{rect}} \]

\[ X_c = \frac{1}{A} \int_{x=0}^{x=b} x dA = \frac{1}{A} \left[ \int_{x=0}^{x=a} x dA + \int_{x=a}^{x=b} x dA \right] = \frac{1}{A} \left[ (A_x)_{\text{tri}} + (A_x)_{\text{rect}} \right] \]
Composite body

\[ A = \sum_{i=1}^{n} A_i \]

\[ Xc = \frac{1}{A} \sum_{i=1}^{n} Xc_i A_i \]
Composite body

\[ A = A_{tri} + A_{rect} - A_{cir} \]

\[ Xc = \frac{1}{A} \left[ (XcA)_{tri} + (XcA)_{rect} - (XcA)_{cir} \right] \]
Example

Locate the centroid of the composite area shown in the Figure.
<table>
<thead>
<tr>
<th></th>
<th>Shape</th>
<th>Area</th>
<th>X-bar</th>
<th>My</th>
<th>Y-bar</th>
<th>Mx</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Square</td>
<td>(7 \times 7 = 49.00)</td>
<td>-(a/2 = 3.5)</td>
<td>-171.5</td>
<td>(a/2 = 3.5)</td>
<td>171.5</td>
</tr>
<tr>
<td>2</td>
<td>Quadrant</td>
<td>(\pi r^2/4 = 38.48)</td>
<td>(4r/3\pi = 2.97)</td>
<td>114.33</td>
<td>(4r/3\pi = 2.97)</td>
<td>114.33</td>
</tr>
<tr>
<td>3</td>
<td>Square</td>
<td>(7 \times 7 = 49.00)</td>
<td>(a/2 = 3.5)</td>
<td>171.5</td>
<td>-(a/2 = -3.5)</td>
<td>-171.5</td>
</tr>
<tr>
<td>4</td>
<td>Triangle</td>
<td>(1/2 \times 7 \times 7 = 24.50)</td>
<td>-(a/3 = -2.33)</td>
<td>-57.17</td>
<td>-(a/3 = 2.33)</td>
<td>-57.17</td>
</tr>
<tr>
<td>5</td>
<td>Circle</td>
<td>(-\pi r^2 = -28.27)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td>132.7</td>
<td><strong>Xc = 0.431</strong></td>
<td>57.17</td>
<td><strong>Yc = 0.431</strong></td>
<td>57.17</td>
</tr>
</tbody>
</table>