NAME:  Solution  

RIN:  

Wednesday, November 12, 2014
8:00 – 9:50

Please state clearly all assumptions made in order for full credit to be given.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Problem #1 (25)

A grand piano lid is supported by two hinges at $A$ and $B$ as well as a prop stick $CD$. The lid weighs 50 lb and the center-of-weight is at $G$. The two hinges are properly aligned therefore exert only force reactions on the lid. Only the hinge at $A$ reacts to axial forces, if any. $L = 40$ in, $l = 26$ in, $a = 22$ in, $b = 17$ in, $c = 11$ in, and $\alpha = 50^\circ$.

a) Complete the free-body-diagram for the lid in the given picture on the right. (7)
b) Express each force labeled in Part a) in Cartesian vector form. (4)
c) Express the moment of all forces in part b) about $A$ in Cartesian vector form. (6)
d) Write scalar equations-of-equilibrium for the lid. (6)
e) Solve magnitude of reaction by prop stick $CD$ and reaction components at $A$ and $B$. (2)

Solution:

(1) FBD (shown above) each reaction force is 1 point; xyz coordinate system and weight is 0.5 point each.

(2) each force component is 0.5 point

$A = A_x i + A_y j + A_z k$ lb

$B = B_y j + B_z k$ lb

$F = F_{CD}(-\cos 50\, j + \sin 50\, k) = F_{CD}(-0.643\, j + 0.766\, k)$ lb

$W = -50k$ lb

(3) each moment is 2 points
\[
\mathbf{r}_{AB} = -L \mathbf{i} = -40 \mathbf{i} \text{ in}
\]
\[
\mathbf{M}_B = \mathbf{r}_{AB} \times \mathbf{B} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-40 & 0 & 0 \\
0 & B_y & B_z
\end{vmatrix} = -40 B_y \mathbf{k} + 40 B_z \mathbf{j} \text{ lb-in}
\]

\[
\mathbf{r}_{AE} = -c \mathbf{i} + a \mathbf{j} = -11 \mathbf{i} + 22 \mathbf{j} \text{ in}
\]
\[
\mathbf{M}_G = \mathbf{r}_{AE} \times \mathbf{W} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-11 & 22 & 0 \\
0 & 0 & -50
\end{vmatrix} = -22 \times 50 \mathbf{i} - 11 \times 50 \mathbf{j} = -1100 \mathbf{i} - 550 \mathbf{j} \text{ lb-in}
\]

\[
\mathbf{r}_{AC} = (a + b + l \cos \theta) \mathbf{j} = (22 + 17 + 26 \cos 50) \mathbf{j} = 55.7 \mathbf{j} \text{ in}
\]
\[
\mathbf{M}_D = \mathbf{r}_{AC} \times \mathbf{F}_{CD} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & 55.7 & 0 \\
0 & -0.642 & 0.766
\end{vmatrix} F_{CD} = 55.7 \times 0.766 F_{CD} \mathbf{i} = 42.7 F_{CD} \mathbf{i} \text{ lb-in}
\]

\begin{align*}
\sum F_x &= A_x = 0 \quad (1) \\
\sum F_y &= A_y + B_y - 0.643 F_{CD} = 0 \quad (2) \\
\sum F_z &= A_z + B_z + 0.766 F_{CD} - 50 = 0 \quad (3) \\
\sum M_x &= 42.7 F_{CD} - 1100 = 0 \quad (4) \\
\sum M_y &= 40 B_z - 550 = 0 \quad (5) \\
\sum M_z &= -40 B_y = 0 \quad (6)
\end{align*}

(5) each none-zero force is 0.5 point.

From (4) \[ F_{CD} = \frac{1100}{42.7} = 25.8 \text{ lb} \]

From (5) \[ B_z = \frac{550}{40} = 13.8 \text{ lb} \quad \text{and from (6) } B_y = 0. \]

From (1) \[ A_x = 0. \] From (2):

\[ A_y = 0.643 \times 25.8 - 0 = 16.6 \text{ lb} \]

From (3) \[ A_z = 50 - 13.8 - 0.766 \times 25.8 = 16.4 \text{ lb} \]

Summary: \[ F_{CD} = 25.8 \text{ lb}, A_x = 0, A_y = 16.6 \text{ lb}, A_z = 16.4 \text{ lb}, B_y = 0, \text{ and } B_z = 13.8 \text{ lb}. \]
**Problem #2 (25)**

The I-beam shown is subjected to a distributed load as well as a concentrated load.

a) Determine the magnitude of a concentrated load equivalent to the triangular distributed load. (8)

b) Determine the location of the concentrated load found in part (a) measured from the left end of the beam (12)

c) Use your results to calculate the reactions at both supports. The support A is a smooth pin, and the support at B is a smooth roller. (5)

**Note:** Neglect the weight of the I-beam and draw the FBD for part c.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Load (kN)</th>
<th>x-bar (m)</th>
<th>My (kN.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.8</td>
<td>1.2</td>
<td>2.16</td>
</tr>
<tr>
<td>2</td>
<td>1.2</td>
<td>2.2</td>
<td>2.64</td>
</tr>
<tr>
<td>Total</td>
<td>3.0</td>
<td></td>
<td>4.8</td>
</tr>
</tbody>
</table>

From the table we get:

a) Magnitude of the concentrated load = 3.0 kN
b) Location from left end = 4.8 / 3.0 = 1.6 m

c) \( \sum M_A = 0 \)

\(-3 \times 0.4 - 1.5 \cos(30) \times 3.6 + 4.8B_y = 0 \Rightarrow B_y = 1.224 \text{ kN} \uparrow \)

\( \sum F_x = 0 \Rightarrow A_x - 1.5 \sin(30) = 0, A_x = 0.75 \text{ kN} \rightarrow \)

\( \sum F_y = 0 \Rightarrow Ay - 3 - 1.5 \cos(30) + 1.224 = 0, A_y = 3.07 \text{ kN} \uparrow \)
Problem #3 (25)

The truss shown in the figure is supported by a smooth pin at A and a roller at F.

a) Identify zero-force members, if any. (2)

b) Use method-of-joint to determine forces in AB, AG, CD, and DE. (13)

c) Use method-of-section to determine forces in BC, CF, and EF. (10)

In b) and c), you must show FBDs, indicate if the force is tension (T) or compression (C), and use the specified method to get the credit.

Solution:

a) zero-force members: BG and BF. Each is 1 point.
a) FBD for D (1 point) as shown
\[ \sum F_y = 0: F_{CD}\sin30 - 1500 = 0 \Rightarrow F_{CD} = 3000 \text{ lb (T)} \] (2 points)
\[ \sum F_x = 0: -3000\cos30 - F_{DE} = 0 \Rightarrow F_{DE} = -2598 \text{ lb} \Rightarrow F_{DE} = 2598 \text{ lb (C)} \] (2 points)
Support reaction for A: FBD (1 point) as shown.
\[ \sum F_x = 0: \Rightarrow A_x = 0. \] (1 point)
\[ \sum M_A = 0: \Rightarrow a A_y - 2500a - 1500(2a) = 0. \Rightarrow A_y = 5500 \text{ lb } \downarrow. \] (1 point)
FBD for A (1 point) as shown
\[ \sum F_y = 0: -F_{AB}\cos60 - F_{AG}\cos30 - 5500 = 0 \] (1 point)
\[ \sum F_x = 0: F_{AB}\sin60 + F_{AG}\sin30 = 0 \] (1 point)
\[ F_{AG} = -9527 \text{ lb} = 9527 \text{ lb (C)} \] (1 point)
\[ F_{AB} = 5500 \text{ lb (T)} \] (1 point)

b) FBD for section CDE is shown (4 points)
\[ \sum M_C = 0: -4 F_{EF} - (8\cos30)1500 = 0. \] (1 point)
\[ \Rightarrow F_{EF} = -2598 \text{ lb} = 2598 \text{ lb (C)}. \] (1 point)
\[ \sum M_E = 0: a F_{BC} - 2500a - 1500(2a) = 0. \] (1 point)
\[ \Rightarrow F_{BC} = 5500 \text{ lb (T)}. \] (1 point)
\[ \sum F_x = 0: 2598 - 5500\cos30 - F_{CF}\cos30 = 0. \] (1 point)
\[ \Rightarrow F_{CF} = -2500 \text{ lb} = 2500 \text{ lb (C)} \] (1 point)
**Problem #4 (25)**

For the shown matrix:

a) Using the method of cofactor expansion calculate the determinant of matrix \( A \) \( \text{(10)} \)

b) Let matrix B be the intersection of rows ‘1 through 3’ and columns ‘2 through 4’ (shaded area). Show how to obtain the inverse of matrix B by using elementary row operations. To receive full credit, all elementary row operations shall be identified and each result shown. You may combine up to two elementary row operations in a single step as long as you clearly identify both operations. \( \text{(15)} \)

\[
A = \begin{bmatrix}
0 & -1 & 2 & 0 & 5 \\
2 & 1 & -3 & 0 & -1 \\
-3 & 5 & 4 & 3 & 2 \\
0 & 2 & -3 & 0 & 6 \\
0 & -2 & 3 & 0 & -5 
\end{bmatrix}
\]

a) Expanding Column 3 : 1

\[
\det(A) = (-1)^{3+4} m_{3,4} = -3
\]

Expanding row 3: \((-3)(-1)^{2+1} m_{1,2} = (-3)(-2)\)

Expanding row 1: \(6[-1(15 - 18) - 2(-10 + 12) + 5(6 - 6)] = -6\)

\[
\begin{pmatrix}
-1 & 2 & 0 & 1 & 0 & 0 \\
1 & -3 & 0 & 0 & 1 & 0 \\
5 & 4 & 3 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & 0 & -3 & -2 & 0 \\
0 & 1 & 0 & -1 & -1 & 0 \\
0 & 0 & 3 & 19 & 14 & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
-3 & -2 & 0 \\
-1 & -1 & 0 \\
19/3 & 14/3 & 1/3
\end{pmatrix}
\]