NAME: Solution Section: __________

RIN: ______________________________

Wednesday, November 19, 2014
5:00 – 6:50

Please state clearly all assumptions made in order for full credit to be given.

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Problem #1 (25)

The plate shown in the figure weighs 150 lb. and is supported in a horizontal position by two hinges and a cable CD. The hinges have been properly aligned; therefore, they exert only force reactions on the plate. Assume that the hinge at B resists any forces along the axis of the hinges.

a) Draw free-body-diagram for the plate.

b) Express all forces in Cartesian vector form.

c) Expression moment of all forces about B in Cartesian vector form.

d) Write scalar equations-of-equilibrium for the plate.

e) Solve magnitude of the cable tension and components at A and B.

Solution: (a) FBD: each reaction force is 1 point; xyz coordinate system and weight is 0.5 point each.

(b) each force component is 0.5 point

\[ \mathbf{A} = A_x \mathbf{i} + A_z \mathbf{k} \text{ lb} \]

\[ \mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k} \text{ lb} \]

\[ \mathbf{F} = F_{CD} \mathbf{u}_{CD} = F_{CD} \frac{22 \mathbf{i} + 22 \mathbf{j} + 18 \mathbf{k}}{\sqrt{22^2 + 22^2 + 18^2}} = F_{CD} (0.612 \mathbf{i} + 0.612 \mathbf{j} + 0.5 \mathbf{k}) \text{ lb} \]

\[ \mathbf{W} = 150 \mathbf{k} \text{ lb} \]
(c) each moment is 2 points

\( \mathbf{r}_{BA} = 24 \mathbf{i} \) in

\[ \mathbf{M}_A = \mathbf{r}_{BA} \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \end{pmatrix} = 24 \mathbf{A}_y \mathbf{k} \quad 24 \mathbf{A}_z \mathbf{j} \ \text{lb-in} \]

\( \mathbf{r}_{BB} = 18 \mathbf{i} + 11 \mathbf{j} \) in

\[ \mathbf{M}_G = \mathbf{r}_{AE} \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \end{pmatrix} = 11 \mathbf{150} \mathbf{i} + 18 \mathbf{150} \mathbf{j} = 1650 \mathbf{i} + 2700 \mathbf{j} \ \text{lb-in} \]

\[ \mathbf{r}_{BC} = 14 \mathbf{i} + 22 \mathbf{j} \) in

\[ \mathbf{M}_D = \mathbf{r}_{AC} \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \end{pmatrix} = 14 \mathbf{14} \mathbf{i} + 22 \mathbf{22} \mathbf{j} + 0.5 \mathbf{F}_{CD} \]

\[ = \mathbf{F}_{CD} (11 \mathbf{i} \ 8.57 \mathbf{k} \ 13.5 \mathbf{k} \ 7 \mathbf{j}) = \mathbf{F}_{CD} (11 \mathbf{i} \ 7 \mathbf{j} \ 22 \mathbf{k}) \ \text{lb-in} \]

(d) each equation is 1 point

\[ F_x = B_x + 0.612 F_{CD} = 0 \quad (1) \]

\[ F_y = A_y + B_y \ 0.612 F_{CD} = 0 \quad (2) \]

\[ F_z = A_z + B_z + 0.5 F_{CD} \quad 150 = 0 \quad (3) \]

\[ M_x = 11 F_{CD} \quad 1650 = 0 \quad (4) \]

\[ M_y = 24 A_z \quad 7 F_{CD} + 2700 = 0 \quad (5) \]

\[ M_z = 24 A_y \quad 22 F_{CD} = 0 \quad (6) \]

(e)

From (4) \( F_{CD} = 150 \text{ lb.} \)
From (1) \( B_x = -91.8 \text{ lb.} \)
From (5) \( A_z = 68.75 \text{ lb.} \)
From (6) \( A_y = 137.5 \text{ lb.} \)
From (2) \( B_y = -45.7 \text{ lb.} \)
From (3) \( B_z = 6.25 \text{ lb.} \)
Problem #2 (25)

Determine the coordinates of the centroid of the shaded area.

Note: You need to show your work to receive credit.

Use a model 1:10 for simplicity, Remember to multiply the answer by 10 at the end.

Use the centroid coordinates:

\[ x_c = 10 \times \frac{771.3}{87} = 88.7 \text{ mm} \]
\[ y_c = 10 \times \frac{326.6}{87} = 37.5 \text{ mm} \]
Problem #3 (25)

The truss shown in the figure is supported by a smooth pin at I and a roller at A.

a) Identify zero-force members, if any. (2)

b) Use method-of-joint to determine forces in DE, EF, AI, and IH. (13)

c) Use method-of-section to determine forces in GH, BG, and BC. (10)

In c) through d), you must show FBDs, indicate if the force is tension (T) or compression (C), and use the specified method to get the credit.

(a) Zero forces: DF, CF Each is 1 point.

(b) FBD of E (1 point) as shown

\[ \sum F_y = 0: \ F_{EF}\sin 20 - 40 = 0 \Rightarrow F_{EF} = 117 \text{ kN (T)} \] (2 points)
\[ \sum F_x = 0: -117\cos 20 - F_{DE} = 0 \Rightarrow F_{DE} = -12.9 \text{ kN} \Rightarrow F_{DE} = 110 \text{ kN (C)} \] (2 points)

FBD of entire truss (0.5 point) as shown to get reactions at A and I.

\[ \sum M_I = A_x\times 4\tan 20 - 20 \times 2 - 40 \times 4 = 0: A_x = 137 \text{ kN} \] 0.5 point
\[ \sum F_x = 137 + I_x = 0: I_x = -137 \text{ kN} = 137 \text{ kN} \] ← 1 point
\[ \sum F_y = I_y - 20 - 40 = 0: I_y = 60 \text{ kN} \] 1 point

FBD of I (1 point) as shown.

\[ \sum F_x = 0: F_{II}\cos 20 - 137 = 0 \] (1 point),
\[ \sum F_y = 0: -F_{II}\sin 20 - F_{AI} + 60 = 0 \] (1 point)
\[ F_{AG} = 146 \text{ kN (T)} \] (1 point)
\[ F_{AI} = 10 \text{ kN (T)} \] (1 point)

(c) FBD (4 points) of the section CEF as shown

\[ \sum M_{II} = 0: -2\tan 20 F_{BC} - 2\times 40 = 0. \] (1 point)
\[ \Rightarrow F_{BC} = -110 \text{ kN} = 110 \text{ kN (C)} \] (1 point)
\[ \sum M_B = 0: 3\sin 20 F_{GH} - 20 \times 1 - 40 \times 3 = 0. \] (1 point)
\[ \Rightarrow F_{GH} = 136 \text{ kN (T)} \] (1 point)
\[ \alpha = \tan^{-1}(2\tan 20/1) = 36.05^\circ \]
\[ \Sigma F_x = 0: 110 - 136 \cos 20 - F_{BG} \cos 36.05 = 0. \quad (1 \text{ point}) \]

\[ \Rightarrow F_{BG} = -22 \text{ kN} = 22 \text{ kN} \quad (C) \quad (1 \text{ point}) \]

FBD for the entire truss

FBD for joint E

FBD for joint I

FBD for section CEG

\[ \Sigma F_y = 0: 60 - 137 = 20 \text{ kN} \quad (C) \quad (1 \text{ point}) \]

\[ \Rightarrow F_{AI} = 137 \text{ kN} \quad (C) \quad (1 \text{ point}) \]

\[ \Rightarrow F_{Hi} = 60 \text{ kN} \quad (C) \quad (1 \text{ point}) \]
Problem #4 (25)

For the shown matrix:

a) Using the method of cofactor expansion calculate the determinant of matrix $A$.

$$A = \begin{bmatrix} 6 & -2 & 3 & -1 & -4 \\ 0 & -5 & 6 & 0 & 0 \\ 4 & 3 & -1 & -2 & -5 \\ 0 & 0 & 1 & 0 & 0 \\ 2 & 3 & -2 & -1 & 5 \end{bmatrix}$$

$$\text{det}(A) = (-1)^{4+3} m_{3,4} = -1 \begin{vmatrix} 6 & -2 & -1 & -4 \\ 0 & -5 & 0 & 0 \\ 4 & 3 & -2 & -5 \\ 2 & 3 & -1 & 5 \end{vmatrix}$$

Expanding row 4:

$$\begin{vmatrix} 6 & -2 & -1 & -4 \\ 0 & -5 & 0 & 0 \\ 4 & 3 & -2 & -5 \\ 2 & 3 & -1 & 5 \end{vmatrix}$$

Expanding row 2:

$$\begin{vmatrix} 6 & -2 & -1 & -4 \\ 0 & -5 & 0 & 0 \\ 4 & 3 & -2 & -5 \\ 2 & 3 & -1 & 5 \end{vmatrix}$$

Expanding row 1:

$$5[6(-10 - 5) + 1(20 + 10) - 4(-4 + 4)] = 300$$

b) Let matrix $B$ be the intersection of rows ‘2 through 4’ and columns ‘2 through 4’ (shaded area).

Show how to obtain the inverse of matrix $B$ by using elementary row operations. To receive full credit, all elementary row operations shall be identified and each result shown. You may combine up to two elementary row operations in a single step as long as you clearly identify both operations.

$$B = \begin{bmatrix} 1 & -1 & -2 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1.2 & 0 & -2 & 0 \\ 0 & -4.6 & -2 & .6 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

To find the inverse of $B$, we perform the following operations:

1. $-1/5 \times R_2$
2. Switch $R_2$ and $R_3$
3. $-1.2R_2$
4. $-2R_1$
5. $4.6R_2$
6. $-1.2R_1$
7. $2R_1$
8. $A^{-1} = \begin{bmatrix} -2 & 0 & -1.2 \\ 0 & 0 & 1 \\ -0.3 & -0.5 & -2.3 \end{bmatrix}$