Please state clearly all assumptions made in order for full credit to be given.

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**Problem #1 (25)**

Three forces are applied on a particle as shown in the figure.

(a) (10) Express forces $F_1$, $F_2$, and $F_3$ in Cartesian vector form.

(b) (10) Determine the magnitude $|R|$, in N, and the angles $\theta_x$, $\theta_y$ and $\theta_z$ in degrees, between the resultant $R$ of the three forces and the axes $x$, $y$ and $z$.

(c) (5) Is the particle in equilibrium?

\[
F_1 = 1000 \cos 145 \cos 35 \mathbf{i} + \sin 145 \cos 35 \mathbf{j} + \sin 35 \mathbf{k} \text{ (N)}
\]

\[
F_2 = 600 (\frac{2}{24^{1/2}} \mathbf{i} - \frac{4}{24^{1/2}} \mathbf{j} + \frac{2}{24^{1/2}} \mathbf{k}) = 244.9 \mathbf{i} - 489.9 \mathbf{j} + 244.9 \mathbf{k} \text{ (N)}
\]

\[
F_3 = -250 \mathbf{k} \text{ (N)}
\]

(b) \[
R = (-671.0 + 244.9) \mathbf{i} + (469.8 - 489.9) \mathbf{j} + (573.6 + 244.9 - 250) \mathbf{k} \text{ N} =
\]

\[
= (- 426.1 \mathbf{i} - 20.1 \mathbf{j} + 568.5 \mathbf{k}) \text{ N}
\]

\[
|R| = (426.1^2 + 20.1^2 + 568.5^2)^{1/2} = 710.8 \text{ N}
\]

\[
\cos \theta_x = (-426.1) / 710.8 = -0.599 ; \theta_x = 126.8^\circ
\]

\[
\cos \theta_y = (-20.1) / 710.8 = -0.028 ; \theta_y = 91.6^\circ
\]

\[
\cos \theta_z = 568.5 / 710.8 = 0.7998 ; \theta_z = 36.9^\circ
\]

(c) Since $R \neq 0$; the particle is not in equilibrium.
Problem #2 (25)

Consider the two vectors, A and B, given below:

\[ A = -3 \mathbf{i} + 4 \mathbf{j} + 10 \mathbf{k} \]
\[ B = 5 \mathbf{i} - 3 \mathbf{j} + 8 \mathbf{k} \]

(a) (5) Determine the unit vector, \( \mathbf{u}_A \).
(b) (5) Determine the unit vector, \( \mathbf{u}_B \).
(c) (5) Determine the angle, \( \theta \), between vectors A and B. Provide your result in degrees.
(d) (5) Determine the vector component of B along A.
(e) (5) Determine the vector component of B orthogonal to A.

(a) \( A = -3 \mathbf{i} + 4 \mathbf{j} + 10 \mathbf{k} \)
\[
3^2 + 4^2 + 10^2 = 125
\]
\[ \mathbf{u}_A = (-3 \mathbf{i} + 4 \mathbf{j} + 10 \mathbf{k})/125^{1/2} = -0.2683 \mathbf{i} + 0.3578 \mathbf{j} + 0.8944 \mathbf{k} \]

(b) \( B = 5 \mathbf{i} - 3 \mathbf{j} + 8 \mathbf{k} \)
\[
5^2 + 3^2 + 8^2 = 98
\]
\[ \mathbf{u}_B = (5 \mathbf{i} - 3 \mathbf{j} + 8 \mathbf{k})/98^{1/2} = 0.5051 \mathbf{i} - 0.3030 \mathbf{j} + 0.8081 \mathbf{k} \]

(c) \( \cos \theta = \mathbf{u}_A \cdot \mathbf{u}_B = -0.2683*0.5051 + 0.3578*(-0.3030) +0.8944*0.8081 = 0.4788 \)
\[ \theta = 61.4^\circ \]

(d) \( B_{\parallel A} = (||B| \cos \theta) \mathbf{u}_A = 98^{1/2}*0.4788 (-0.2683 \mathbf{i} + 0.3578 \mathbf{j} + 0.8944 \mathbf{k}) = \]
\[ = -1.272 \mathbf{i} + 1.696 \mathbf{j} + 4.240 \mathbf{k} \]

(e) \( B_{\perp A} = B - B_{\parallel A} = (5 -1.272) \mathbf{i} + (-3 -1.696) \mathbf{j} + (10 - 4.240) \mathbf{k} \)
\[ = 6.272 \mathbf{i} - 4.696085 \mathbf{j} + 3.760 \mathbf{k} \]
Problem #3 (25)

A 100 kg mass M is suspended by cable AM (length 0.4 m) from a steel ring at A. Cable AC (length 1.0 m) connects the ring to a rigid ceiling at an angle θ from vertical, as shown. Cable AB (length 0.7 m) is pulled with a force F at an angle φ from horizontal, as shown. The acceleration due to gravity is 9.81 m/s².

Let “T” be the tension in cable AC. Let “W” be the tension in cable AM.

The three cables are attached to ring A in such a way that they are free to slide around the ring. Thus, the lines of action of the cables are concurrent at the center of the ring.

a (6). Draw a complete and separate free body diagram for the forces acting on the ring at A.

b (10). Write all of the applicable equations of static equilibrium for the motionless ring at A.

c (2). List all of the unknowns in the system of equations in Part b.

d (2). Can someone solve the system of equations in Part b for the unknowns in Part c? Explain why or why not?

Now suppose that cable CAB is one continuous cable that goes through the smooth ring at A, and is not attached to the ring.

e (2). How do the equations of static equilibrium change from Part b?

f (3). If the magnitude and direction of the pulling force (F, at an angle φ from the horizontal) are specified, can someone always solve the system of equations in Part e? Why or why not?

a) See figure at right.

+1 FOR LABELED AXES (ALL OR NOTHING)
+1 FOR LABELED ANGLES (ALL OR NOTHING; ANY TWO ANGLES CAN BE IDENTIFIED IF EQUIVALENT TO WHAT IS AT RIGHT)
+1 FOR LABELED FORCES (ALL OR NOTHING; 981 N INSTEAD OF W IS ACCEPTABLE)
+1 FOR EACH CORRECTLY FACING ARROW = +3 TOTAL

b)  

\[ T \cos \theta - \phi + F \cos 180 - \phi = 0 \]

\[ T \cos \theta + F \cos 90 - \phi - mg = 0 \]
or

\[ T \sin \theta - F \cos \varphi = 0 \]
\[ T \cos \theta + F \sin \varphi = mg \]

+2 FOR EACH NON-ZERO TERM (ALL OR NOTHING), INCLUDING ITS SIGN; +10 TOTAL

\[ mg \text{ AS } 981 \text{ N IS ACCEPTABLE, AS LONG AS THE } - \text{ SIGN IS SHOWN.} \]

c)  F, T, \theta, \varphi

+2 FOR ALL FOUR UNKNOWNS, ALL OR NOTHING
d)  No, the system cannot be solved because there are more unknowns (4) than equations (2)

+2 FOR ANYTHING INDICATING MORE UNKNOWNS THAN EQUATIONS
e)  Change T to F

+2 FOR ANYTHING INDICATING THE TENSION T SIMPLY BECOMES F

f)  +3 FOR ANYTHING INDICATING THAT THERE WILL BE TWO EQUATIONS BUT ONLY ONE
UNKNOW (\theta), SO THE SYSTEM IS OVER-DETERMINED, SO IT CANNOT ALWAYS BE SOLVED.
Problem #4 (25)

The following are three linear equations in three unknowns (x₁, x₂, and x₃). Solve the system of equations using the Gauss-Jordan elimination method. Be careful to observe the signs (+ and −) in the problem statement.

\[-5x₁ - 2x₂ + 2x₃ = -2\]
\[2x₁ - 2x₃ = 14\]
\[x₁ + \frac{1}{2}x₂ - \frac{1}{2}x₃ = 6\]

A (3). Show the augmented matrix for the system shown above.

B (18). Show how to obtain the reduced row echelon form of the augmented matrix by using elementary row operations. To receive full credit, all elementary row operations shall be identified and each result shown. You may combine up to two elementary row operations in a single step as long as you clearly identify both operations.

C (4). Write the solution for the three variables x₁, x₂, and x₃.

Note: To receive full credit, all intermediate work (e.g. elementary row operations should be shown.)

A)

\[
\begin{array}{ccc|c}
-5 & -2 & 2 & -2 \\
2 & 0 & -2 & 14 \\
1 & \frac{1}{2} & -\frac{1}{2} & 6 \\
\end{array}
\]

+1 POINT FOR EACH CORRECT ROW

Any permutation of the three rows is permitted; a vertical dotted line between the third and fourth columns is permitted.

B)

+4 FOR AN EXAMPLE OF EACH TYPE OF ROW OPERATION PERFORMED CORRECTLY (SWAP ROWS, MULTIPLY/DIVIDE ROW, TRANSFORM ONE ROW USING ANOTHER), UP TO +12. IF ONLY TWO OPERATIONS USED (E.G., NO ROW SWAPS), THEN +6 EACH, UP TO +12.

i) divide row two by 2; multiply row three by 2:

\[
\begin{array}{ccc|c}
-5 & -2 & 2 & -2 \\
1 & 0 & -1 & 7 \\
2 & 1 & -1 & 12 \\
\end{array}
\]

ii) swap rows one, two, and three to bring “1” into the (1,1) position

\[
\begin{array}{ccc|c}
1 & 0 & -1 & 7 \\
2 & 1 & -1 & 12 \\
-5 & -2 & 2 & -2 \\
\end{array}
\]

iii) elimination phase – clear the (2,1) and (3,1) elements:

multiply row one by (−2) and add it to row two
multiply row one by (5) and add it to row three

\[
\begin{array}{ccc}
1 & 0 & -1 & 7 \\
0 & 1 & 1 & -2 \\
0 & -2 & -3 & 33 \\
\end{array}
\]

iv) clear the (3,2) element:
multiply row two by (2) and add it to row three

\[
\begin{array}{ccc}
1 & 0 & -1 & 7 \\
0 & 1 & 1 & -2 \\
0 & 0 & -1 & 29 \\
\end{array}
\]

v) multiply row three by (−1)

\[
\begin{array}{ccc}
1 & 0 & -1 & 7 \\
0 & 1 & 1 & -2 \\
0 & 0 & 1 & -29 \\
\end{array}
\]

vi) back substitution phase – clear the (1,3) and (2,3) elements:
multiply row three by (1) and add it to row one
multiply row three by (−1) and add it to row two

\[
\begin{array}{ccc}
1 & 0 & 0 & -22 \\
0 & 1 & 0 & 27 \\
0 & 0 & 1 & -29 \\
\end{array}
\]

**+6 IF THE ABOVE MATRIX IS OBTAINED EXACTLY**

**+3 IF WRONG BUT THE STUDENT STATES IT’S WRONG AFTER CHECKING THEIR WORK (SEE BELOW)**

C)
The solution is obtained by inspection, in x₁, x₂, x₃ order:

\[
\begin{align*}
x₁ &= -22 \\
x₂ &= 27 \\
x₃ &= -29 \\
\end{align*}
\]

**+4 IF THE SOLUTION MATCHES THE STUDENT’S RESULT IN STEP B-vi, ABOVE – ALL OR NOTHING.**

CHECK

\[
\begin{align*}
-5(-22) - 2(27) + 2(-29) &= -2 & \checkmark \\
2(-22) - 2(-29) &= 14 & \checkmark \\
(-22) + (1/2)(27) - (1/2)(−29) &= 6 & \checkmark \\
\end{align*}
\]