NAME: Solution

RIN: ________________________________

Section: __________

Wednesday, December 12, 2012

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
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<tr>
<td>2</td>
<td>20</td>
<td></td>
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<tr>
<td>3</td>
<td>20</td>
<td></td>
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<td>4</td>
<td>20</td>
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<tr>
<td>5</td>
<td>20</td>
<td></td>
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<tr>
<td>6</td>
<td>20</td>
<td></td>
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<tr>
<td>Total</td>
<td>100</td>
<td></td>
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</tbody>
</table>

**N.B.:** You will be graded on 5 problems, 20 points per problem. Problems 1, 2, and 3 are mandatory and will be graded. Before turning in your exam, please make sure you have circled the two problems you want to be graded out of problems 4, 5 and 6.

Good Luck
Problem 1 (20 points)

The truss structure shown is supported at A by a smooth pin and at E by a smooth roller.

(a) Determine the reactions at supports A and E  
(b) Using the Method of Sections, determine the forces in members $BC, CG$ and $FG$, and state whether the member is in tension or compression  
(c) Using the Method of Joints, determine the force in member $AB$, and state whether the member is in tension or compression  

Note: You have to draw all needed FBD in (a), (b) and (c) to get full credit.
\( \Sigma M_C = -(400)(10) + (T_{FG})(10) = 0 \)
\( T_{FG} = \frac{4000}{5} = 800 \text{ lb} \)
\( T_{FC} = 800 \text{ lb} \) (4)
\( \Sigma M_E = -(400)(10) - (T_{BC})(10) = 0 \)
\( T_{BC} = -\frac{4000}{5} = -800 \text{ lb} \)
\( T_{EC} = 400 \text{ lb} \) (5)
\( \Sigma M_F = -(400)(10) - (-400)(10) \)
\(-T_{EC}(\cos 45^\circ)(10) = 0 \)
\( T_{EC} = -\frac{4000}{10\cos 45^\circ} = -566 \text{ lb} \)
\( T_{CC} = 566 \text{ lb} \) (5)
\( \Sigma F_x = T_{AC} \cos 45^\circ = 0 \)
\( T_{AC} = 0 \)
\( \Sigma F_y = T_{AB} + 900 = 0 \)
\( T_{AB} = -900 \text{ lb} \)
\( T_{AB} = 900 \text{ lb} \) (6)
Problem 2 (20 points)

Given the system of linear equations:

\[ \begin{align*}
3x + y - 2z &= 3 \\
4x + y - 4z &= 1 \\
2x + 3y + z &= 2
\end{align*} \]

a. Calculate the \( \text{det}(A) \) using the method of cofactors by expanding the 3rd row.  (3 pts.)
b. Calculate the \( \text{Adj}(A) \).  (5 pts.)
c. Determine \( A^3 \).  (3 pts.)
d. Use Cramer's rule to solve for the three unknowns.  (9 pts.)

\[ A = \begin{bmatrix} 3 & 1 & -2 \\ y & 1 & -y \\ 2 & 3 & 1 \end{bmatrix} \]

\[ \text{m}_{11} = \begin{vmatrix} 1 & -y \\ 2 & 1 \end{vmatrix} = 13 \quad \text{m}_{12} = \begin{vmatrix} y & -y \\ 2 & 1 \end{vmatrix} = 12 \]

\[ m_{13} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 10 \quad m_{21} = \begin{vmatrix} 1 & -y \\ 3 & 1 \end{vmatrix} = 7 \]

\[ m_{22} = \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} = 7 \quad m_{23} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 7 \quad m_{31} = \begin{vmatrix} y & -y \\ 3 & 1 \end{vmatrix} = -2 \quad m_{32} = \begin{vmatrix} 1 & -y \\ 3 & 1 \end{vmatrix} = -2 \]

\[ M = \begin{bmatrix} 13 & 12 & 10 \\ 7 & 7 & 7 \\ -2 & -y & -1 \end{bmatrix} \]

\[ C = \begin{bmatrix} 13 & -2 & -2 \\ -2 & -y & -1 \\ -12 & 7 & -1 \end{bmatrix} \quad \text{det}(A) = 2(-2) + 3 \cdot 4 + 1(-1) = -4 + 12 - 1 = \frac{7}{2} \]

b) \( \text{adj}(A) = C^T = \begin{bmatrix} 13 & -2 & -2 \\ -2 & -y & -1 \\ -12 & 7 & -1 \end{bmatrix} \)

c) \( A^{-1} = \frac{\text{adj}(A)}{\text{det}(A)} = \begin{bmatrix} 13/7 & -1 & -2/7 \\ -11/7 & 1 & 4/7 \\ 10/7 & -1 & -1/7 \end{bmatrix} \)

d) \( A_1 = \begin{bmatrix} 3 & 1 & -2 \\ 1 & -y & -4 \\ 2 & 3 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 3 & 3 & -2 \\ 4 & 1 & -4 \\ 2 & 3 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 3 & 1 & 3 \\ 4 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix} \)

\[ x_1 = \frac{\text{det}(A_1)}{\text{det}(A)} \quad x_2 = \frac{\text{det}(A_2)}{\text{det}(A)} \quad x_3 = \frac{\text{det}(A_3)}{\text{det}(A)} \]

\[ \text{exp 1st column: det}(A_1) = 3 \cdot 13 + 1(-7) + 1(-1) = 39 - 7 - 4 = 28 \quad x_1 = \frac{28}{7} = \frac{4}{1} \]

\[ \text{exp 2nd column: det}(A_2) = 3 \cdot (-12) + 1 \cdot 7 + 2 \cdot 4 = -36 + 7 + 8 = -21 \quad x_2 = \frac{-21}{7} = -3 \]

\[ \text{exp 3rd column: det}(A_3) = 2 \cdot 10 + 1(-7) + 6(-1) = 30 - 7 - 6 = 21 \quad x_3 = \frac{21}{7} = \frac{3}{1} \]
A rope is attached to the top corner of a block A, passes over a fixed drum, and is attached to another block as shown in the Figure below. The weight of block A is 400-N and the coefficient of friction between this block and the floor is 0.30. The coefficient of friction between the rope and the drum is 0.60.

Determine the maximum mass of block B for which no motion occurs. Will the motion correspond to sliding or tipping of block A.

SOLUTION
Free-body diagram for block A for impending slipping: (2 pt)

\[
A_f = 0.30A_n \quad (1 pt)
\]

\[
\Sigma F_x = T \cos 20^\circ - A_f = 0 \quad (1 pt)
\]

\[
T = 1.0642A_f = 1.0642 (0.30A_n) = 0.3193A_n \]

\[
\Sigma F_y = T \sin 20^\circ + A_n - 400 = 0 \quad (1 pt)
\]

\[
A_n = 360.62 \text{ N} \]
\[ A_f = 0.30A_n = 0.30(360.62) = 108.19 \text{N} \]

\[ T = 1.0642A_f = 1.0642(108.19 \text{N}) = 115.13 \text{ N} \]
\[ T_{\text{tip}} = 115.13 \text{ N} \] (1 pt)

Free-body diagram for impending tipping:

\[ \Sigma M = 400(0.25) - T \cos 20^\circ (1) - T \sin 20^\circ (0.5) = 0 \] (1 pt)
\[ T = 90.033 \] (1 pt)
\[ T_{\text{tip}} = 90.033 < 115.13 \text{ N} \text{ (block will tip before it slips)} \] (2 pt)

For the fixed drum:

\[ T_1 = T_{\text{tip}} \] (2 pt)

\[ \beta = (2\pi)(90+20)/360 = 1.9199 \]
\[ \mu \beta = 0.60(1.9199) = 1.1519 \]
\[ T_2 = T_1 e^{\mu \beta} = 90.033 e^{1.1519} = 284.873 \text{ N} \] (1 pt)
\[ W_B = T_2 = 284.887 \text{ N} \]
\[ M_B = W_B / 9.81 = 284.873 \text{ N} / 9.81 = 29.041 \text{ kg} \text{ Maximum mass} \] (1 pt)

Block will tip before it slips (2 pt)
Problem 4 (20 points)

A frame ABC is supported in part by cable DBE that passes through a frictionless ring at B. If the tension in that cable is 385N, determine the magnitude and the unit vector of the resultant of the forces exerted by the cable at B.

\[
B = (480,0,600),\ D=(0,510,280),\ E=(210,400,0)
\]

\[
BD = D-B = (-480,510,-320)
\]

\[
T_{BD} = \frac{T_{BD} \cdot (-480,510,-320)}{\|BD\|} = \frac{T_{BD} \cdot (-48,51,-32)}{\sqrt{48^2 + 51^2 + 32^2}}
\]

\[
= 385 \frac{(-48,51,-32)}{77} = (-240i + 255j - 160k) \text{N}
\]

\[
BE = E-B = (-270,510,-320)
\]

\[
T_{BE} = \frac{T_{BE} \cdot (-270,400,-600)}{\|BE\|} = \frac{T_{BD} \cdot (-27,40,-60)}{\sqrt{27^2 + 40^2 + 60^2}}
\]

\[
= 385 \frac{(-27,40,-60)}{77} = (-135i + 200j - 300k) \text{N}
\]

\[
R = T_{BD} + T_{BE} = (-375i + 455j - 460k) \text{N}
\]

\[
R = \sqrt{375^2 + 455^2 + 460^2} = 747.83 \text{N}
\]

\[
\lambda_R = \frac{(-375i + 455j - 460k)}{747.83} = (-.50i + .60j + .615k)
\]
Collars A and B are connected by a 25-inch-long wire and can slide freely on frictionless rods. The collars provide forces that resist motion perpendicular to the rods they slide on. A 60-lb force \( Q \) is applied to collar B in the positive z direction as shown. An unknown force \( P \), applied to collar A in the positive x direction, keeps the system in equilibrium. The distance \( z = 9 \) inches.

a) Draw free-body diagrams for the collars A and B. Treat the collars as particles. (6)

b) Using the given geometry, find the distance \( z \). (1)

c) Express the force acting on collar B due to the wire as an unknown magnitude times a unit vector. (3)

d) Write the equations of equilibrium for collar B and use them to find the tension in the wire. (5)

e) Write the equations of equilibrium for collar A and use them to find the magnitude of the force \( P \). (5)

\[ \begin{align*}
\text{Collar A:} \quad N_x^A & \quad \text{for axis} \quad \hat{\mathbf{x}} \quad \hat{\mathbf{y}} \quad \hat{\mathbf{z}} \\
N_y^A & \quad \text{for including normal forces} \\
T \hat{\mathbf{z}}_{AB} & \quad \text{for including \( Q, P, T \) forces}
\end{align*} \]

\[ \begin{align*}
b) \quad \overrightarrow{AB} &= \overrightarrow{B} - \overrightarrow{A} = (0, 0, 2) - (-9, 20, 0) = (-9, 20, 2) \\
&= (-9m, 20m, 2) \\
&= 81 \text{ m}^2 + 400 \text{ m}^2 + 2^2 = 625 \text{ m}^2 \Rightarrow z = 12 \text{ in}
\end{align*} \]

\[ \begin{align*}
c) \quad -\lambda_{AB} &= \frac{\overrightarrow{AB}}{11AB} = \frac{(9, 20, 2)}{25} \text{ m} = \left( \frac{9}{25}, \frac{4}{5}, \frac{2}{25} \right) \\
\text{or wrong sign} \quad 3 \times (1) \text{ for each correct value of unit vector}
\end{align*} \]

\[ \begin{align*}
d) \quad \sum \overrightarrow{F}_B &= \overrightarrow{Q} \hat{\mathbf{k}} + T \left( \frac{9}{25} \hat{\mathbf{i}} + \frac{4}{5} \hat{\mathbf{j}} - \frac{2}{25} \hat{\mathbf{k}} \right) + N_x^B \hat{\mathbf{i}} + N_y^B \hat{\mathbf{j}} = \overrightarrow{0}
\end{align*} \]

\[ \begin{align*}
&\therefore \quad Q - \frac{12}{25} T = 0 \\
&\Rightarrow \quad T = 25 \cdot 60 \text{ lb} \\
&= 1250 \text{ lb}
\end{align*} \]

\[ \begin{align*}
e) \quad \sum \overrightarrow{F}_A &= P \hat{\mathbf{i}} - T \left( \frac{9}{25} \hat{\mathbf{i}} + \frac{4}{5} \hat{\mathbf{j}} - \frac{2}{25} \hat{\mathbf{k}} \right) + N_x^A \hat{\mathbf{j}} + N_y^A \hat{\mathbf{k}} = \overrightarrow{0}
\end{align*} \]

\[ \begin{align*}
&\therefore \quad P - \frac{T q}{25} = 0 \\
&\Rightarrow \quad P = \frac{9}{25} T \\
&= 45 \cdot 16 \text{ lb}
\end{align*} \]
The beam AB supports two concentrated loads and rests on soil that exerts a linearly distributed upward load as shown in the figure. Determine the values of \(w_A\) and \(w_B\) corresponding to equilibrium.

\[ R_1 = (\omega_A)(1.8) \text{ kN} \]

\[ R_2 = \frac{1}{2}(1.8)(\omega_B - \omega_A) = 0.9(\omega_B - \omega_A) \text{ kN} \]

\[ \sum M_D = 0 \rightarrow -R_1(0.3) - (30)(0.3) + (24)(0.6) = 0. \]

\[ -R_1(0.3) = 9 + 14.4 = 0. \rightarrow (0.54)(\omega_A) = 5.4 \]

\[ \omega_A = 10 \text{ kN/m} \]

\[ \sum F_y = 0 \rightarrow R_1 + R_2 - 24 - 30 = 0. \]

\[ (10)(1.8) + R_2 - 54 = 0. \rightarrow R_2 = 36 \text{ kN} \]

\[ (0.9)(\omega_B - 10) = 36 \rightarrow \omega_B = 50 \text{ kN/m} \]