Please state clearly all assumptions made in order for full credit to be given.

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Problem #1 (25)

Three traffic lights, each weighing 100 lb., are hung at an intersection on the pole assembly as shown. The traffic light at $D$ also sustains a horizontal force due to wind parallel to the x-axis. The base of the pole, $O$, is a fixed support. The weight of the pole assembly can be neglected.

(a) Draw complete and separate free body diagram for the pole assembly (7)
(b) Write down the equilibrium equations for the assembly including ALL detailed force and moment terms (14)
(c) Determine all reactions at the base $O$ in Cartesian vector form (4)

Note: You need to show your work to receive credit.

Solution:

(1) FBD: each force 0.5 point, coordinate system 0.5 point, each moment 1 point. Total 7 points.

\[ \sum M_O = M_{ox}i + M_{oy}j + M_{oz}k + 100 \times 25j + 100 \times 50j + r_{OD} \times -100k - 75i = 0 \]  (1)
Selection of point O: 0.5 point; each moment and force component (totally 7 terms) 0.5 points; \( r_{OD} \), 25 ft, and 50 ft each 0.5 points.

\[
\sum R = O_x i + O_y j + O_z k - 300k - 75i = 0 \quad (2)
\]

Each force term (totally 7) 0.5 point.

where \( r_{OD} = 50k + 35j \) ft

1 point - students can earn this here if they write it down in the next step.

(3) From (1)

\[
\sum M_O = M_{ox}l + M_{oy}j + M_{oz}k + 2500j + 5000j + (50k + 35j) \times (-100k - 75i) = 0
\]

\[
\sum M_O = M_{ox}l + M_{ey}j + M_{oz}k + 7500j - 3500l - 3750j + 2625k = 0
\]

\( M_{ox} - 3500 = 0 \Rightarrow M_{ox} = 3500 \text{ lb-ft} \) (1 point)
\( M_{oy} + 7500 - 3750 = 0 \Rightarrow M_{oy} = -3750 \text{ lb-ft} \) (1 point)
\( M_{oz} + 2625 = 0 \Rightarrow M_{oz} = -2625 \text{ lb-ft} \) (1 point)

\[
M_O = 3500l - 3750j - 2625k \text{ lb-ft} \) (1 point)

From (2):

\( O_x - 75 = 0 \Rightarrow O_x = 75 \text{ lb}; \) (1 point)
\( O_y = 0 \text{ lb}; \) (1 point)
\( O_z - 300 = 0 \Rightarrow O_z = 300 \text{ lb}; \) (1 point)

\( O = 75l + 300k \text{ lb} \) (1 point)
Problem #2 (25)

There is a distributed load across a 10’ long weightless beam supported at the left end by a smooth pin and at the right end by a smooth roller. The weight on the beam per linear foot [lb/ft] is characterized by a function as follows:

\[
\begin{align*}
0 < x < 2 & : 0 \\
2 < x < 6 & : (x - 2)^2 \\
6 < x < 10 & : 16
\end{align*}
\]

Where \( x = 0 \) corresponds to the left end of the beam.

What is the centroid of the weight distribution along the x-axis? (12)
What are the support reactions at \( x=0' \) and \( x=10' \)? (8)
(Show all work to maximize credit).

Evaluate as three different single forces:

\[ W_A = 0 \]
\[ W_B = \int_0^2 f(x) \, dx = \frac{4^3}{3} \approx 21.33 \]
\[ W_C = 4 \cdot 16 = 64 \]

Find centroids to determine moment arms of moments:

\[ M_A = 0 \]
\[ \text{distance from } x=2 \text{ to centroid of } B = \frac{\int_0^2 x^3 \, dx}{\int_0^2 x^2 \, dx} = \frac{4^4}{4^3} = \frac{21.33}{21.33} = 1 \]
\[ \text{weight } B \text{ is equivalent } 21.33 \text{ lb} \text{ acting at } x=3' \]
\[ \text{distance to center from } x=6 \]
\[ d = \frac{12}{2} = 6' \]
Support reactions:

\[ F_0 + F_{10} = 64 + 21.33 = 85.33 \]

\[ \sum M_0 = 0 \]

\[ (21.33 \times 5) + (64 \times 8) = 10 \times F_{10} \]

\[ 106.65 + 512 = F_{10} = 61.87 \]

\[ F_0 = 85.33 - 61.87 = 23.46 \]

Find Centroid: 1) Distribution

<table>
<thead>
<tr>
<th>Region</th>
<th>Area (in²)</th>
<th>Centroid</th>
<th>Area \cdot Centroid</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>64/3</td>
<td>5</td>
<td>106.67</td>
</tr>
<tr>
<td>C</td>
<td>64</td>
<td>8</td>
<td>512.00</td>
</tr>
</tbody>
</table>

\[ \bar{y} = \frac{\sum \text{Area} \cdot \text{Centroid}}{\sum \text{Area}} = \frac{618.67}{85.33} \approx 7.25 \]
Problem #3 (25)

Determine the force in each of the 7 left-side members (including DE) of the Pratt truss illustrated below. Determine if they are in compression (C) or tension (T).

- Support reaction (y-direction) at A (2)
- Support reaction (x-direction) at A (2)
- AB (3)
- AC (3)
- BC (3)
- BE (3)
- BD (3)
- CE (3)
- DE (3)

You must show all steps and required FBD’s. You may use any method you like (Joints or Sections) in your solution.
**SOLUTION**

Free body: Truss:

\[ \Sigma F_y = 0: \quad A_y = 0 \]

\[ \Sigma M_a = 0: \quad 9(H - 18 ft) \times (-4 \text{kips})(9 \text{ft}) - 9(H - 18 ft) \times (-4 \text{kips})(27 \text{ft}) = 0 \]

\[ H = 6 \text{kips} \]

\[ \Sigma F_y = 0: \quad A_j = 6 \text{kips} - 12 \text{kips} = 0 \quad A_y = 6 \text{kips} \]

Free body: Joint A:

\[ \Sigma F_y = 0: \quad F_{AB} + F_{AC} = \frac{F_{AM}}{3} = 6 \text{kips} \]

\[ F_{AB} = 7.50 \text{kips} \quad F_{AC} = 4.50 \text{kips} \]

Free body: Joint C:

\[ \Sigma F_x = 0: \quad F_{CE} = 4.50 \text{kips} \quad F_{DC} = 4.00 \text{kips} \]

\[ \Sigma F_y = 0: \quad F_{EC} = 4.50 \text{kips} \]

Free body: Joint B:

\[ \Sigma F_y = 0: \quad -\frac{4}{3} F_{EB} + \frac{4}{3} (7.50 \text{kips}) - 400 \text{kips} = 0 \]

\[ F_{EB} = 2.50 \text{kips} \]

\[ F_{EB} = -6.00 \text{kips} \quad F_{EB} = 6.00 \text{kips} \]

Free body: Joint D:

We note that DE is a zero-force member:

\[ F_{DC} = 0 \]

Also,

\[ F_{DF} = 5.00 \text{kips} \]

From symmetry:

\[ F_{EF} = F_{DE} \quad F_{DF} = 2.50 \text{kips} \]

\[ F_{DC} = F_{CE} \quad F_{EC} = 4.50 \text{kips} \]

\[ F_{DC} = F_{EC} \quad F_{EC} = 4.00 \text{kips} \]

\[ F_{DF} = 6.00 \text{kips} \]

\[ F_{DE} = F_{CE} \quad F_{CE} = 7.50 \text{kips} \]

\[ F_{DE} = F_{DC} \]

\[ F_{DE} = 5.00 \text{kips} \]

**NOT REQUIRED IN TEST PROBLEM**
Problem #4 (25)

Consider the three equations

\[\begin{align*}
  x_1 - 2x_3 &= -1 \\
  -2x_1 + x_2 + 6x_3 &= 7 \\
  3x_1 - 2x_2 - 5x_3 &= -3
\end{align*}\]

(a) Write the equation \(Ax=b\) in parametric matrix form where \(A\) is the associated coefficient matrix and \(b\) is the solution matrix to the equations.

(b) Given the associated coefficient matrix \(A\), find its determinant using cofactor expansion along row #2.

(c) Determine \(A^{-1}\), the inverse of \(A\), using only row operations.

(d) Utilizing \(A^{-1}\), solve the system of equations, for the variables \(x_1, x_2,\) and \(x_3\).

Show all work.

a)

\[
\begin{bmatrix}
1 & 0 & -2 \\
-2 & 1 & 6 \\
3 & -2 & -5
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= 
\begin{bmatrix}
-1 \\
7 \\
-3
\end{bmatrix}
\]

b) \(c_{2,1} = -[0 \times (-5) - (-2) \times (-2)] = 4\)

\(c_{2,2} = 1 \times (-5) - (-2) \times (3) = -5 + 6 = 1\)

\(c_{2,3} = -[1 \times (-2) - 0 \times 3] = 2\)

\(\det(A) = 4 \times -2 + 1 \times 1 + 2 \times 6 = 5\)

c) 

\[
\begin{bmatrix}
1 & 0 & -2 & 1 & 0 & 0 \\
-2 & 1 & 6 & 0 & 1 & 0 \\
3 & -2 & -5 & 0 & 0 & 1
\end{bmatrix}
\]

Add \(2R_1\) to \(R_2\); subtract \(3R_1\) from \(R_3\)

\[
\begin{bmatrix}
1 & 0 & -2 & 1 & 0 & 0 \\
0 & 1 & 2 & 2 & 1 & 0 \\
0 & -2 & -1 & -3 & 0 & 1
\end{bmatrix}
\]

Add \(2R_2\) to \(R_3\)

\[
\begin{bmatrix}
1 & 0 & -2 & 1 & 0 & 0 \\
0 & 1 & 2 & 2 & 1 & 0 \\
0 & 0 & 5 & 1 & 2 & 1
\end{bmatrix}
\]

Divide \(R_3\) by 5

\[
\begin{bmatrix}
1 & 0 & -2 & 1 & 0 & 0 \\
0 & 1 & 2 & 2 & 1 & 0 \\
0 & 0 & 1 & .2 & .4 & .2
\end{bmatrix}
\]

Add \(2R_3\) to \(R_1\) and subtract \(2R_3\) from \(R_2\)
\[
\begin{bmatrix}
1 & 0 & 0 & 1.4 & 0.8 & 0.4 \\
0 & 1 & 0 & 1.6 & 0.2 & -0.4 \\
0 & 0 & 1 & 0.2 & 0.4 & 0.2 \\
\end{bmatrix}
\]

\[A^{-1} = \begin{bmatrix}
1.4 & 0.8 & 0.4 \\
1.6 & 0.2 & -0.4 \\
0.2 & 0.4 & 0.2 \\
\end{bmatrix}\]

\[
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} = \begin{bmatrix}
1.4 & 0.8 & 0.4 \\
1.6 & 0.2 & -0.4 \\
0.2 & 0.4 & 0.2 \\
\end{bmatrix}^{-1} \begin{bmatrix}
-1 \\
7 \\
-3 \\
\end{bmatrix}
\]

\[x = 1.4 \times (-1) + 0.8 \times 7 + 0.4 \times (-3) = -1.4 + 5.6 - 1.2 = 3\]

\[y = 1.6 \times (-1) + 0.2 \times 7 - 0.4 \times (-3) = -1.6 + 1.4 + 1.2 = 1\]

\[z = 0.2 \times (-1) + 0.4 \times 7 + 0.2 \times (-3) = -0.2 + 2.8 - 0.6 = 2\]