Please state clearly all assumptions made in order for full credit to be given.

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Problem #1 (25)

Consider the following four matrices A, B, C, and D,

\[
A = \begin{bmatrix} 3 & 3 & 1 \\ 0 & -3 & 2 \\ 3 & 5 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 4 & 5 \\ -1 & 3 & 2 \\ -3 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 0 \\ 2 & 5 \\ -3 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}
\]

Determine the following:

a) \( BA \)  

b) \( 3C - 2B^t \)  

c) \( C + D \)  

d) \( \text{Det}(A) \) using the duplicate column method  

e) \( D^{-1} \)  

Note: You need to show your work to receive credit.

\begin{align*}
\text{a) } & \quad (\text{9(4)})(7) \quad (\text{9(2)})(11) \quad (\text{8(4)})(6) \\
& \quad \begin{bmatrix} 9 & 7 & 11 \end{bmatrix} \quad \begin{bmatrix} 3 & -2 & 7 \end{bmatrix} \\
\text{b) } & \quad \begin{bmatrix} 12 & 0 \\ 6 & 15 \\ -9 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -8 & -6 \\ -10 & -4 \end{bmatrix} \\
& \quad \begin{bmatrix} 16 & 2 \\ -2 & 9 \\ -19 & -7 \end{bmatrix} \\
\text{c) } & \quad \text{C + D is undefined as matrices have different sizes} \\
\text{d) } & \quad \text{Det}(A) = \begin{vmatrix} 3 & 3 & 1 \\ 0 & -3 & 2 \\ 3 & 5 & 1 \end{vmatrix} \\
& \quad = [(-9+18+0)-(-9+30+0)] = -12 \\
\text{e) } & \quad \begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}, \quad \text{Det}(D) = \begin{vmatrix} (12) - (2) \end{vmatrix} = 10 \\
& \quad \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.4 \end{bmatrix}
\end{align*}
**Problem #2 (25)**

A transmission tower is held by three cables attached to a pin at A and anchored by bolts at B, C, and D. If an upward vertical force of 2100 lb. was exerted by the tower on the pin at A:

a) Draw a complete and separate FBD of the particle in this problem  
(5)
b) Express all forces in your FBD in Cartesian Vector Form  
(12)
c) Write down the equations of equilibrium  
(5)
d) Solve the equations of equilibrium to determine the tension in each cable  
(3)

![Diagram of the transmission tower with cables and pins labeled A, B, C, and D. The pin at A is connected to the cables, and the forces are shown in Cartesian vector forms.](image)

**Solution:**

1. Draw free body diagram at the Pin A (5 points):
2. Express the forces in Cartesian vector forms (12 points):

The coodination:
A (0, 90, 0); B (-45, 0, 30); C (30, 0, 65); D (20, 0, -60)  
(2 points)

\[ \vec{AB} = -45\text{i} - 90\text{j} + 30\text{k}; \quad AB = 105 \text{ ft} \]  
(1 point)

\[ \vec{AC} = 30\text{i} - 90\text{j} + 65\text{k}; \quad AC = 115 \text{ ft} \]  
(1 point)

\[ \vec{AD} = 20\text{i} - 90\text{j} - 60\text{k}; \quad AD = 110 \text{ ft} \]  
(1 point)
\[T_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{AB}{AB} = (-\frac{3}{7} \mathbf{i} - \frac{6}{7} \mathbf{j} + \frac{2}{7} \mathbf{k})T_{AB}\]  
\[T_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{AC}{AC} = (\frac{6}{23} \mathbf{i} - \frac{18}{23} \mathbf{j} + \frac{13}{23} \mathbf{k})T_{AC}\]  
\[T_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{AD}{AD} = (\frac{2}{11} \mathbf{i} - \frac{9}{11} \mathbf{j} - \frac{6}{11} \mathbf{k})T_{AD}\]  

\[P = 2100 \mathbf{j}\]  

(3) 3D equilibrium conditions  
\[\Sigma F = 0: T_{AB} + T_{AC} + T_{AD} + P = 0\]  
\[\Sigma F_x = 0; \Sigma F_y = 0; \Sigma F_z = 0\]  

\[-\frac{3}{7} T_{AB} + \frac{6}{23} T_{AC} + \frac{2}{11} T_{AD} = 0\]  
\[-\frac{6}{7} T_{AB} - \frac{18}{23} T_{AC} - \frac{9}{11} T_{AD} + 2100 \text{ lb} = 0\]  
\[\frac{2}{7} T_{AB} + \frac{13}{23} T_{AC} - \frac{6}{11} T_{AD} = 0\]  

Solving the system of linear equations using conventional algorithms give:  
\[T_{AB} = 841.55 \text{ lb}\]  
\[T_{AC} = 624.38 \text{ lb}\]  
\[T_{AD} = 1087.81 \text{ lb}\]  

(3 point)
Problem #3 (25)

1. A force $F$ is applied at point $C$ of the bent bar shown below. The magnitude of the force is 120 lb. Leg OA is in the x-z plane. Leg AB is parallel to the y-axis, and leg BC is vertically up parallel to the z-axis.

2. Express the force $F$ in a Cartesian vector form. \hspace{1cm} (4)

3. Determine the moment of the force about point $O$. Express the moment in a Cartesian vector form. \hspace{1cm} (6)

4. Determine the scalar component of the moment about the line $OA$. \hspace{1cm} (5)

5. Express the moment along the line $OA$ in Cartesian vector form \hspace{1cm} (5)

6. Determine the vector component of the moment orthogonal to line $OA$. \hspace{1cm} (5)

\[ F = F_{eF} = F_{eCD} = 120 \ e_{CD} \text{ lb} \]

\[ e_{CD} = \frac{CD}{||CD||} = \frac{-3i - 14j + 5k}{((-3)^2 + (-14)^2 + (5)^2)^{\frac{1}{2}}} \]

\[ e_{CD} = -0.2i - 0.92j + 0.33k \]

\[ F_{CD} = -23.74i - 110.78j + 39.56k \]

\[ M_{O} = r_{OC} \times F_{CD} \]

\[ r_{OC} = 20.00i + 22.00j + 32.00k \]

\[ M_{O} = r_{OC} \times F_{CD} \]

\[ M_{O} = \begin{vmatrix} i & j & k \\ +20.00 & +22.00 & +32.00 \\ -23.74 & -110.78 & +39.56 \end{vmatrix} = i ((22.00)x(39.56)-(110.78)x(32.00)) - j ((20.00)x(39.56)-(23.74)x(32.00)) + k ((20.00)x(-110.78)-(23.74)x(22.00)) \]

\[ = 4415.28i - 1550.88j - 1693.32k \]

\[ e_{OA} = \frac{OA}{||OA||} = \frac{(20i + 0j + 22k)}{(20^2 + 0^2 + 22^2)^{\frac{1}{2}}} \]

\[ e_{OA} = 0.67i + 0j + 0.74k \]

\[ M_{OC} = M_{O} \cdot e_{OA} = (0.67)(4415.28) + (0)(-1550.88) + (0.74)(-1693.32) = 1705.18 \text{ lb in} \]

\[ M_{OC} = M_{OC} \cdot e_{OA} = 1705.18 \times (0.67i + 0j + 0.74k) = (1142.47i + 0j + 1261.83k) \text{ lb in} \]
\[ M_{\perp} = M_0 - M_{OC} = (4415.28i - 1550.88j - 1693.32k) - (1142.47i + 0j + 1261.83k) \\
= (3272.81i - 1550.88j - 2955.15k) \]
Problem #4 (25)

A beam $ABCD$ is loaded and supported as shown below. The support at $A$ is a frictionless pin. The cable $BED$ passes over a frictionless pulley at $E$. The beam has a uniform cross section and a mass of 20 kg located at $B$. The force $F$ has a magnitude of 500 N.

1. Draw a complete free body diagram of the beam (separate from the figure below). Clearly label all forces acting on the beam. (10)
2. Write down the force equilibrium equations for the beam. (6)
3. Determine the tension in the cable. (4)
4. Determine the reaction forces at $A$. Express the reaction at $A$ in a Cartesian vector form. (5)

![Diagram of the beam with labeled forces and dimensions]

**Equilibrium equations**

$\Sigma M_A = 0$

$-(W)(0.8) - (F)(1.2) + (T \sin 30)(0.8) + (T)(1.6) = 0$

$-(W)(0.8) - (F)(1.2) + T(2) = 0$

$\Sigma F_x = 0$

$A_x + T \cos 30 = 0$
\[ \Sigma F_y = 0 \]
\[ A_y + T \sin 30^\circ + T - W - F = 0 \]

**Tension in the cable:**
Using \( \Sigma M_A = 0 \) or \(-20 \times 9.8 \times 0.8 - 500 \times 1.2 + T(2) = 0\) gives \( T = 378.4 \) N

**Reaction forces at A:**
Using \( \Sigma F_x = 0 \) gives \( A_x = -T \cos 30^\circ = -327.70 \) N
Using \( \Sigma F_y = 0 \) gives
\[
A_y = -T \sin 30^\circ - T + W + F = 128.4 \text{ N}
\]
\[
\Delta = (-327.70 \text{ i} + 128.4 \text{ j}) \text{ N}
\]