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Problem 1 (25 Points)

a) Find the inverse of the matrix $A$, where $A$ is:

$$
A = \begin{bmatrix}
1 & 0 & 3 \\
2 & 0 & 1 \\
0 & 2 & 4 \\
4 & 0 & 0
\end{bmatrix}
$$

$A$ is not invertible since it is not a square matrix.

b) Given the following system of equations:

$$
\begin{align*}
x + z &= 15 \\
y + z &= 12 \\
x + y &= 7
\end{align*}
$$

b$_1$) Write the above system of equations in the form $AX = B$, identifying each of the matrices $A$, $X$ and respectively $B$.

b$_2$) Find $A^{-1}$, the inverse of matrix $A$.

b$_3$) Solve the system of equations by matrix inversion method (i.e. using matrix $A^{-1}$).

**Note:** You need to show all work to receive full credit.

b$_1$) $A = \begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 1 \\
1 & 1 & 0
\end{bmatrix}$, $X = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}$, $B = \begin{bmatrix}
15 \\
12 \\
7
\end{bmatrix}$

b$_2$) $A^{-1} = \begin{bmatrix}
\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\
-\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}$

b$_3$) $X = A^{-1}B = \begin{bmatrix}
\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\
-\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}\begin{bmatrix}
15 \\
12 \\
7
\end{bmatrix} = \begin{bmatrix}
5 \\
2 \\
10
\end{bmatrix}$

$x = 5$, $y = 2$, $z = 10$
Solutions:

Question 1:

\( \mathbf{b} \)  
\[ \bar{T}_{BD} = \frac{\bar{r}_D - \bar{r}_B}{\| \bar{r}_D - \bar{r}_B \|} \]
\[ = \bar{T}_{BD} \left( -\frac{2}{3} \hat{i} + \frac{1}{3} \hat{j} - \frac{2}{3} \hat{k} \right) \]  
(2 pts)

\[ \bar{T}_{EC} = \frac{\bar{r}_C - \bar{r}_E}{\| \bar{r}_C - \bar{r}_E \|} \]
\[ = \bar{T}_{EC} \left( -\frac{6}{7} \hat{i} + \frac{3}{7} \hat{j} + \frac{2}{7} \hat{k} \right) \]  
(2 pts)

\[ \sum \bar{F} = \bar{A} + \bar{T}_{BD} + \bar{T}_{EC} - (270 \text{ lb}) \hat{j} = 0 \]
\[ \bar{i} : \quad A_x - \frac{2}{3} T_{BD} - \frac{6}{7} T_{EC} = 0 \]  
(2)

\[ \bar{j} : \quad A_y + \frac{1}{3} T_{BD} + \frac{3}{7} T_{EC} - 270 \text{ lb} = 0 \]  
(2)

\[ \bar{k} : \quad A_z - \frac{2}{3} T_{BD} + \frac{2}{7} T_{EC} = 0 \]  
(2)

\[ \sum \bar{M}_A = \bar{r}_B \times \bar{T}_{BD} + \bar{r}_E \times \bar{T}_{EC} + (4 \text{ ft}) \bar{i} \times (-270 \text{ lb}) \hat{j} = 0 \]
\[ \bar{j} : \quad 5.333 T_{BD} - 1.714 T_{EC} = 0 \]  
(2)

\[ \bar{k} : \quad 2.667 T_{BD} + 2.571 T_{EC} - 1080 \text{ lb} = 0 \]  
(2)

Solve the 5 equations for the 5 unknowns,

\[ T_{BD} = 101.3 \text{ lb} \quad T_{EC} = 315 \text{ lb} \]  
(2)

\[ \bar{A} = (338 \text{ lb}) \bar{i} + (101.2 \text{ lb}) \hat{j} - (22.5 \text{ lb}) \hat{k} \]  
(2)

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5 points if they mix up X and Z.
Problem 3 (25 Points)

The truss shown is supported at A by a smooth pin and at G by a rocker.

a) Using the method of joints, determine the force in kN in members CD and DE and state whether the members are in tension or compression.

b) Using the method of sections, determine the force in members BC, BE, and EF and state whether they are in tension or compression.

c) Identify any zero-force member you can find

Note: You have to draw all needed FBD and the part of the truss you are using in b to get full credit.

---

**a) FBD of J + D**

\[ \Sigma F_y = 0 \]

\[ -14 - \text{DE } \sin \theta = 0 \quad -14 - \text{DE } \frac{2}{\sqrt{13}} = 0 \quad \text{DE} = -7\sqrt{13} \text{ kN} = 7\sqrt{13} \text{ kN} \]

\[ \Sigma F_x = 0 \quad -\text{CD} - \text{DE } \cos \theta = 0 \]

\[ -\text{CD} - (-7\sqrt{13}) \left( \frac{3}{\sqrt{13}} \right) = 0 \]

\[ -\text{CD} + 21 = 0 \quad \text{CD} = 21 \text{ kN} \]

**b) Considering the RHS of the truss**

\[ \Sigma M_B = 0 \]

\[ -14 (3) - EF \sin \alpha (3) - EF \cos \alpha (2) = 0 \]

\[ -8\sqrt{10} - EF \frac{8}{\sqrt{10}} - EF \frac{6}{\sqrt{10}} = 0 \]

\[ EF = -\frac{8\sqrt{10}}{9} \text{ kN} = -29.51 \text{ kN} = 29.51 \text{ kN} \]

\[ \Sigma M_C = 0 \quad 14 (3) + BC (2) = 0 \quad BC = 21 \text{ kN} \]

**c) CE and AG**
Solution Problem 4

1. FBD of whole system:
   \[ \sum M_B = 0.5Ax - (1.9)(784.8) = 0 \]
   \[ 0.5Ax - 1491.1 = 0 \]
   \[ Ax = 2982.2 \text{ N} \]

   \[ \sum F_x = Ax + Bx = 0 \]
   \[ Bx = -Ax = -2982.2 \text{ N} \]
   \[ Bx = 2982.2 \text{ N} \]

2. FBD of pulley:
   \[ T = 784.8 \text{ N} \]

3. FBD of member BC
   \[ \sum M_C = -(1)By + (0.7)(784.8) = 0 \]
   \[ By = 549.4 \text{ N} \]

   \[ \sum F_x = Cx - 2982.2 = 0 \]
   \[ Cx = 2982.2 \text{ N} \]

   \[ \sum F_y = By + Cy + 784.8 = 0 \]
   \[ 549.4 + Cy + 784.8 = 0 \]
   \[ Cy = -1334.2 \text{ N} \]

4. From (1):
   \[ \sum F_y = Ay + By - w = 0 \]
   \[ Ay + 549.4 - 784.8 = 0 \]
   \[ Ay = 235.4 \text{ N} \]
Solution Problem 4

FBD of whole system:
\[ EM = 0.5 Ax - (1.9)(784.8) \]
\[ 0.5 Ax - 1491.1 = 0 \]
\[ Ax = 2982.2 \text{ N} \]
\[ EF_x = Ax + Bx = 0 \]
\[ Bx = -Ax = -2982.2 \text{ N} \]
\[ Bx = 2982.2 \text{ N} \]

FBD of pulley:

FBD of member BC
\[ n_c = -1(By) + 0.7(784.8) \]
\[ By = 549.4 \text{ N} \]
\[ EF_x = Cx + 2982.2 = 0 \]
\[ Cx = -2982.2 \text{ N} \]
\[ EF_y = By + Cy + 784.8 = 0 \]
\[ Cy = -1334.2 \text{ N} \]

From 5:
\[ EF_y = Ay + By - W = 0 \]
\[ Ay + 549.4 - 784.8 = 0 \]
\[ Ay = 235.4 \text{ N} \]

Grading Key

1. For wrong Ax
2. For wrong Bx
3. For wrong W
4. For wrong T
5. For wrong By
6. For wrong Cx
7. For wrong Cy
8. For wrong Ay