NAME: Solution  
Section: __________

Monday, May 14, 2012

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
</tr>
</tbody>
</table>

**N.B.:** You will be graded on 5 problems, 20 points per problem. Problems 1, 2, and 3 are mandatory and will be graded. Before turning in your exam, please make sure you have circled the two problems you want to be graded out of problems 4, 5 and 6.

**Good Luck**
\[ \begin{bmatrix} 10 & -12 & 3 \\ 0 & 4 & 5 \\ 3 & 4 & 20 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 3 \end{bmatrix} \]

\[ \text{det}(A) = a_{11} C_{11} + a_{21} C_{21} + a_{31} C_{31} \]

\[ \text{any first column} = 10 \cdot (-1)^{1+1} \begin{vmatrix} 4 & 5 \\ 4 & 20 \end{vmatrix} - 3 \cdot (-1)^{3+1} \begin{vmatrix} -12 & 3 \\ 4 & 5 \end{vmatrix} \]

\[ = 10 \times (80 - 20) + 3 \times (-60 - 12) \]

\[ = 10 \times 60 + 3 \times (-72) \]

\[ = 600 - 216 \]

\[ = 384 \]

\[ \text{det}(A) = 384 \]

\[ \text{Zero credit if any other method is used.} \]

\[ C_{11} = (-1)^{1+1} \begin{vmatrix} 4 & 5 \\ 4 & 20 \end{vmatrix} \]

\[ C_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 5 \\ 3 & 20 \end{vmatrix} \]

\[ C_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 4 \\ 3 & 4 \end{vmatrix} \]

\[ C_{21} = (-1)^{2+1} \begin{vmatrix} -12 & 3 \\ 4 & 20 \end{vmatrix} \]

\[ C_{22} = (-1)^{2+2} \begin{vmatrix} 10 & 3 \\ 3 & 20 \end{vmatrix} \]

\[ C_{23} = (-1)^{2+3} \begin{vmatrix} 10 & -12 \\ 3 & 4 \end{vmatrix} \]

\[ C_{31} = (-1)^{3+1} \begin{vmatrix} -12 & 3 \\ 4 & 5 \end{vmatrix} \]

\[ C_{32} = (-1)^{3+2} \begin{vmatrix} 10 & 3 \\ 0 & 5 \end{vmatrix} \]

\[ C_{33} = (-1)^{3+3} \begin{vmatrix} 10 & -12 \\ 0 & 4 \end{vmatrix} \]

\[ \therefore \ C_{11} = (80 - 20) \quad C_{12} = -(0 - 15) \quad C_{13} = (0 - 12) \]

\[ C_{21} = -(120 - 12) \quad C_{22} = (200 - 9) \quad C_{23} = -(40 + 36) \]

\[ C_{31} = -(60 - 12) \quad C_{32} = -(50 - 0) \quad C_{33} = (40 - 0) \]

\[ \therefore \ \text{Cof}(A) = \begin{bmatrix} 60 & 15 & -12 \\ 252 & 191 & -76 \\ -72 & -50 & 40 \end{bmatrix} \]

\[ \text{Adj}(A) = \text{Cof}(A)^T = \begin{bmatrix} 60 & 252 & -72 \\ 15 & 191 & -50 \\ -12 & -76 & 40 \end{bmatrix} \]
(d) $A^{-1} = \frac{\text{Adj}(A)}{\det(A)} = \frac{1}{384} \begin{bmatrix} 60 & 252 & -72 \\ 15 & 191 & -50 \\ -12 & -76 & 40 \end{bmatrix}$

(e) $y = \frac{\det(A_2)}{\det(A)}$

with $\det(A) = 384$

$\det(A_2) = \begin{vmatrix} 10 & 10 & 8 \\ 0 & 0 & 5 \\ 3 & 3 & 20 \end{vmatrix} = 0$

Since 2 columns are identical.

$\therefore y = 0$
Problem: Bucket A and block C are connected by a cable that passed over drum B. Knowing that the coefficients of friction at all surfaces are $\mu_s=0.35$, determine the smallest combined mass $m$ of the bucket and its contents for which block C will

(a) Remain at rest
(b) Start moving up the incline

Draw necessary FBDs to receive full credits (20 points)
SOLUTION

(a) Block C remains at rest: Motion impedes.

Drum:
\[ \frac{T_2}{mg} = e^{\mu \beta} = e^{0.35(2\pi/3)} \]
\[ T_2 = 2.0814mg \]

Block C: Motion impedes.
\[ \sum F = 0: \quad N - m_c g \cos 30^\circ = 0 \]
\[ N = m_c g \cos 30^\circ \]
\[ F = \mu, N = 0.35 m_c g \cos 30^\circ \]
\[ + \sum F = 0: \quad T_2 + F - m_c g \sin 30^\circ = 0 \]
\[ 2.0814mg + 0.35 m_c g \cos 30^\circ - m_c g \sin 30^\circ = 0 \]
\[ 2.0814mg = 0.19689 m_c \]
\[ m = 0.09459 m_c = 0.09459 (100 \text{ kg}) \]
\[ m = 9.46 \text{ kg} \]

(b) Block C: Starts moving up \( \mu_s = 0.35 \)

Drum: Impending motion of cable.
\[ \frac{T_2}{T_1} = e^{\mu_s \beta} \]
\[ \frac{mg}{T_1} = e^{0.35(2\pi/3)} \]
\[ T_1 = \frac{mg}{2.0814} \]
\[ = 0.48045mg \]

Block C: Motion impedes.
\[ + \sum F = 0: \quad N - m_c g \cos 30^\circ \]
\[ N = m_c g \cos 30^\circ \]
\[ F = \mu_s N = 0.35 m_c g \cos 30^\circ \]
\[ + \sum F = 0: \quad T_1 - F - m_c g \sin 30^\circ = 0 \]
\[ 0.48045mg - 0.35 m_c g \cos 30^\circ - 0.5 m_c g = 0 \]
\[ 0.48045m = 0.80311 m_c \]
\[ m = 1.67158 m_c = 1.67158 (100 \text{ kg}) \]
\[ m = 167.2 \text{ kg} \]
Problem 3 (20 points)

The following frame is used to support a block of mass 400 kg. The support at A is a smooth pin and at D is a smooth roller.

a) Determine the reactions at the supports  

b) Determine the components of all the forces acting on member ABCD

\[ \text{Note: You need to draw all required FBD's and show all work to receive full credit} \]
FBD of member ABCD

CE is a CF member, so the force in that member is along CE.

\[ \sum M_C = 0 \]
\[ 4,312(5) - 3,920(2) - 1.5B_x = 0 \quad \boxed{2} \]
\[ B_x = 9,146.7 \text{ N} \quad \boxed{1} \]

\[ \sum F_x = 0 \]
\[ -4,312 + 4,312 + 3,920 + 9,146.7 + \frac{C}{\sqrt{5}} = 0 \quad \boxed{2} \]
\[ C = -14,609 \text{ N} = 14,609 \text{ N} / \sqrt{5} \quad \boxed{1} \]

\[ \sum F_y = 0 \]
\[ 3,920 - 14,609 \left( \frac{1}{\sqrt{5}} \right) + B_y = 0 \quad \boxed{1} \]

\[ B_y = 2,613.3 \text{ N} \quad \boxed{1} \]
a) \[ \text{FBD of cylinder B} \]

\[ \Sigma F_x = -N_1 + W \cos 30^\circ = 0 \]

\[ N_1 = 100 \times 9.807 \times \cos 30^\circ = 849.3 \text{ N} \]

\[ [N_1 = 849.3 \text{ N}] \]

\[ \text{Also, } \Sigma F_y = -N_2 + W \cos 60^\circ = 0 \Rightarrow N_2 = 100 \times 9.807 \times \cos 60^\circ = 490.35 \text{ lb} \]

d) \[ \text{From FBD of cylinder C} \]

\[ \Sigma F_x = -N_2 + W \cos 45^\circ = 0 \]

\[ N_2 = 960.7 \times \cos 45^\circ = 693.46 \text{ N} \]

\[ [N_2 = 693.46 \text{ N}] \]

\[ \text{Also, } \Sigma F_y = -N_C + W \cos 45^\circ = 0 \Rightarrow N_c = 693.46 \text{ N} \]

e) \[ \text{From FBD of cylinder A} \]

\[ \Sigma F_x = -N_L \cos 15^\circ + N_c - N_B \sin 15^\circ + W \cos 45^\circ = 0 \]

\[ N_L \cos 15^\circ = 693.46 - 490.35 \sin 15^\circ - 980.7 \times \cos 45^\circ = 1260 \text{ lb} \]

[\[ N_L = 1304.45 \text{ lb} \]
\[ \Sigma F_y = -N_R - N_L \sin 15^\circ + N_b \cos 45^\circ + W \cos 45^\circ = 0 \quad (1) \]

\[ \Rightarrow N_R = 980.7 \cos 45^\circ + 490.3 \cos 15^\circ - 1309.45 \sin 15^\circ \]

\[ = 829.49 \quad (2) \]

\[ [N_R = 829.49 \text{ N}] \quad (1) \]

\[ N_b = 480.34 \text{ N} \quad (1) \]

\[ N_c = 692.61 \text{ N} \quad (1) \]
Problem 5 (20 points)
Two forces 120 N and 160 N and a couple of 25 N.m are applied to an electric motor as shown. It is required to replace these forces and couple by a single resultant force and a single resulting couple at A
a) Determine the magnitude of the resulting force at A in N (4)
b) Determine the angles between the resultant found in part a above and the three coordinate axes in degrees (6)
c) Determine the magnitude of the resulting couple at A in N.m (4)
d) Determine the angles between the resultant found in part c above and the three coordinate axes in degrees (6)

Note: You should show all your work to receive full credit

\[ F_A = (-120 \hat{x}, -160 \hat{k}) \text{ N} \]
\[ F_A = \sqrt{120^2 + 160^2} = 200 \text{ N} \]
\[ \theta_x = \cos^{-1} \left(-\frac{120}{200}\right) = 126.9^\circ \]
\[ \theta_y = \cos^{-1} 0 = 90^\circ \]
\[ \theta_z = \cos^{-1} \left(-\frac{160}{200}\right) = 143.1^\circ \]
\[ \mathbf{r}_{AC} = (0, 0.7, 0.25) \text{ m} \text{ C is along 120 N Force} \]
\[ \mathbf{r}_{AD} = (0.7, 1, 0) \text{ m} \text{ D along 160N at same \tau of A} \]
\[ C_A = (\mathbf{r}_{AC} \times \mathbf{F}_{120} + \mathbf{r}_{AD} \times \mathbf{F}_{160} + 25 \hat{z}) \text{ N m} \]
\[ = 3 \begin{pmatrix} -3 \\ -8 \\ 1 \end{pmatrix} + 4 \begin{pmatrix} -8 \\ 3 \\ 8 \end{pmatrix} + 25 \hat{z} \]
\[ = (-7 \hat{x} + 9 \hat{y} + 24 \hat{k}) \text{ N m} \]
\[ C_A = \sqrt{7^2 + 9^2 + 24^2} = 26.57 \text{ N m} \]
\[ \theta_x = \cos^{-1} \left(-\frac{7}{26.57}\right) = 105.3^\circ \]
\[ \theta_y = \cos^{-1} 0 = 70.2^\circ \]
\[ \theta_z = \cos^{-1} \left(-\frac{24}{26.57}\right) = 25.4^\circ \]
Problem: Determine (a) the distribution load $w_0$ at the end D of the beam ABCD for which the reaction at B is zero; (b) the corresponding reaction at C (20 points).
SOLUTION

(a)

We have

\[ R_I = \frac{1}{2} (18 \text{ ft})(450 \text{ lb/ft}) = 4050 \text{ lb} \]

\[ R_{II} = \frac{1}{2} (18 \text{ ft})(a_0 \text{ lb/ft}) = 9a_0 \text{ lb} \]

Then

\[ \Sigma M_C = 0: \quad -(44,100 \text{ lb} \cdot \text{ft}) + (10 \text{ ft})(4050 \text{ lb}) + (4 \text{ ft})(9a_0 \text{ lb}) = 0 \]

or

\[ a_0 = 100.0 \text{ lb/ft} \]

(b)

\[ \Sigma F_x = 0: \quad C_x = 0 \]

\[ \Sigma F_y = 0: \quad -4050 \text{ lb} - (9 \times 100) \text{ lb} + C_y = 0 \]

or

\[ C_y = 4950 \text{ lb} \]

\[ C = 4950 \text{ lb} \]