NAME: **Solution**

RIN: 

Wednesday, November 28, 2012
5:00 – 6:50

Please state clearly all assumptions made in order for full credit to be given.

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<th>Problem</th>
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<th>Score</th>
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**Good Luck!**
Problem #1 (25 %)

\[ x_1 + 3x_2 + 5x_3 = 10 \\
-2x_1 + 3x_2 + 3x_3 = -1 \\
4x_1 - x_2 + 2x_3 = 14.5 \]

a) Express the three equations in the form \( AX = B \)  

b) Using the method learned in class determine the inverse of matrix \( A \)  

c) Use the inverse found in b) to solve for \( x_1 \), \( x_2 \), and \( x_3 \).  

d) Using the method of cofactor expansion, determine the determinant of matrix \( A \) by expanding along the third row.

**Note:** You need to show all intermediate work to receive credit. Answers not in the box will not be graded.
\[ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9/7 & -11/7 & -6/7 \\ 16/7 & -18/7 & -13/7 \\ -10/7 & 13/7 & 9/7 \end{bmatrix} \begin{bmatrix} 10 \\ -1 \\ 14.5 \end{bmatrix} \]

\[ x_1 = \frac{9}{7} \times 10 - \frac{11}{7} \times (-1) - \frac{6}{7} \times 14.5 = \frac{90}{7} + \frac{11}{7} - \frac{87}{7} = 2 \]  \hspace{1cm} (2)

\[ x_2 = \frac{16}{7} \times 10 - \frac{18}{7} \times (-1) - \frac{13}{7} \times 14.5 = \frac{160}{7} + \frac{18}{7} - \frac{188.5}{7} = -1.5 \]  \hspace{1cm} (2)

\[ x_3 = -\frac{10}{7} \times 10 + \frac{13}{7} \times (-1) + \frac{9}{7} \times 14.5 = -\frac{100}{7} - \frac{13}{7} + \frac{130.5}{7} = 2.5 \]  \hspace{1cm} (2)

d)  \text{det}(A) = 4 \begin{vmatrix} 3 & 5 \\ 3 & 3 \end{vmatrix} - (-1) \begin{vmatrix} 1 & 5 \\ 2 & 3 \end{vmatrix} + 2 \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} \]  \hspace{1cm} (2)

\[ = 4\left[(3)(-15)\right] + \left[(1)(-10)\right] + 2\left[(1)(-6)\right] \]  \hspace{1cm} (1)

\[ = 4(-6) + 13 + 2(9) \]

\[ = -24 + 13 + 18 = 7 \]  \hspace{1cm} (1)
Problem #2 (25%)  
The assembly shown is welded to the collar A that fits on the vertical pin shown. The pin can exert couples about the x and z axes but does not prevent motion about and along the y axis.

a) Draw the free-body diagram of the entire assembly.

b) Determine tension in each cable as a scalar number and support reactions (forces and moments) at A as vectors in Cartesian coordinates.

<table>
<thead>
<tr>
<th>Tension in cable FC</th>
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<tbody>
<tr>
<td>Tension in cable ED</td>
<td></td>
</tr>
<tr>
<td>Support reaction force vector at A</td>
<td></td>
</tr>
<tr>
<td>Support reaction moment vector at A</td>
<td></td>
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</tbody>
</table>

[Diagram of assembly with labeled dimensions and forces]
Solution Problem 2

\[ \overrightarrow{T_{CF}} = T_{CF} \overrightarrow{\lambda_{CF}} = \frac{-(0.08) \overrightarrow{i} + 10.06 \overrightarrow{j}}{\sqrt{(0.08)^2 + (10.06)^2}} T_{CF} \]
\[ = T_{CF} (-0.8 \overrightarrow{i} - 0.6 \overrightarrow{j}) \]

\[ \overrightarrow{T_{DE}} = T_{DE} \overrightarrow{\lambda_{DE}} = \frac{(0.12) \overrightarrow{j} - (0.09) \overrightarrow{k}}{\sqrt{(0.12)^2 + (0.09)^2}} T_{DE} \]
\[ = T_{DE} (0.8 \overrightarrow{j} - 0.6 \overrightarrow{k}) \]

1. \[ \sum F_x = 0 \Rightarrow A_x - 0.8 T_{CF} = 0. \quad (1) \]
2. \[ \sum F_y = 0 \Rightarrow 0.6 T_{CF} + 0.8 T_{DE} - 480 = 0 \quad (2) \]
3. \[ \sum F_z = 0 \Rightarrow A_z - 0.6 T_{DE} = 0. \quad (3) \]

2. \[ \sum \overrightarrow{M_A} = 0. \Rightarrow M_{Ax} \overrightarrow{i} + M_{Az} \overrightarrow{k} + R_{AD} \times \overrightarrow{T_{DE}} + R_{AC} \times \overrightarrow{T_{CF}} + R_{AC} \times (-480 \overrightarrow{j}) = 0. \quad (1) \]

\[ M_{Ax} \overrightarrow{i} + M_{Az} \overrightarrow{k} + \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ 0.08 & 0 & 0.09 \\ 0 & 0.8 T_{DE} & -0.6 T_{DE} \end{vmatrix} + \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ 0.08 & 0 & 0.135 \\ -0.8 T_{CF} & 0 & 0 - 480 \end{vmatrix} + \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ 0.08 & 0 & 0.09 \\ -0.8 T_{CF} & 0 & 0 \end{vmatrix} = 0 \]

\[ M_{Ax} \overrightarrow{i} + M_{Az} \overrightarrow{k} + (0.09)(0.8) T_{DE} \overrightarrow{j} + (0.08)(0.8) T_{DE} \overrightarrow{k} + (-0.135)(0.6 T_{CF}) \overrightarrow{i} - [(0.135)(0.8 T_{CF}) \overrightarrow{j} + (0.08)(0.6 T_{CF}) \overrightarrow{k} + (480)(0.135) \overrightarrow{i} - (0) \overrightarrow{j} + (-480)(0.08) \overrightarrow{k} = 0 \]
i: \[ M_{Ax} - (0.09)(0.8) T_{DE} - (0.135)(0.6 T_{CF}) + (480)(0.135) = 0 \]  
\[(4)\]

j: \[ (6.08)(0.6) T_{DE} - (0.135)(0.8) T_{CF} = 0 \rightarrow T_{DE} = 2.25 \cdot T_{CF} \]  
\[(5)\]

k: \[ M_{Az} + (6.08)(0.8) T_{DE} + (0.08)(0.6 T_{CF}) + (-480)(0.08) = 0 \]  
\[(6)\]

(2) and (5) → \[ (0.6) T_{CF} + (0.8)(2.25) T_{CF} = 480 \]

\[ T_{CF} = 200.00 N \]  
\[(1)\]

(5) → \[ T_{DE} = 2.25 (200.00) \rightarrow T_{DE} = 450.00 N \]  
\[(1)\]

(3) → \[ A_{z} = (0.6)(450.00) = 270.0 \text{ N} = A_{z} \]  
\[(1)\]

(1) → \[ A_{x} = (0.8)(200.0) = 160.0 \text{ N} = A_{x} \]  
\[(1)\]

\[ A = 160.0 \vec{i} + 270.0 \vec{k} \text{ N} \]  
\[(1)\]

(6) → \[ M_{Az} + (0.08)(0.8)(450) + (0.08)(0.6)(200) - (480)(0.08) = M_{Az} = 0 \]  
\[(1)\]

(4): \[ M_{Ax} - (0.09)(0.8)(450) - (0.135)(0.6)(200) + (480)(0.135) = M_{Ax} = -16.2 \text{ N.m} \]  
\[(1)\]

\[ \rightarrow \vec{M}_{A} = -16.2 \vec{i} \text{ N.m} \]  
\[(1)\]
Problem #3 (25%)  

The beam $AB$ with length $L$ ft is welded at $A$ to the wall as shown in the figure. The point of contact $A$ between the beam and the wall may be considered as a fixed support. A distributed load with the distribution function $w = -(x - L)^2 \frac{Lb}{ft}$ is applied to the beam as shown in the figure, where $L$ is the length of the beam.

a) Find the resultant force vector (in the Cartesian coordinates) of the distributed load and the associated point of application ($x_c$) measured from $A$ as a function of $L$.

<table>
<thead>
<tr>
<th>Resultant force vector</th>
<th>$x_c$</th>
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b) Find and express the reaction forces and moment at $A$ as vectors in the Cartesian coordinates as functions of $L$. (Weight of the beam can be neglected.)

<table>
<thead>
<tr>
<th>Support reaction force vector at $A$</th>
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<tbody>
<tr>
<td>Support reaction moment vector at $A$</td>
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$w = -(x - L)^2 \hspace{1cm} lb/ft$
Solution, Problem 3

a)

\[ R = \int_0^L w \, dx = \int_0^L - (x - L)^2 \, dx = -\int_0^L (x^2 + L^2 - 2xL) \, dx \]

\[ = - \left[ \frac{1}{3}x^3 + L^2x - x^2L \right]_0^L = - \left[ \frac{1}{3}L^3 + L^2 - L^2 \right] = -\frac{L^3}{3} \]

\[ X_c = \frac{\int_0^L wx \, dx}{R} \]

\[ \int_0^L wx \, dx = \int_0^L - (x - L)x \, dx = -\int_0^L (x^2 + L^2 - 2xL) \, dx \]

\[ = - \left[ \frac{1}{4}x^4 + \frac{1}{2}x^2 - \frac{2}{3}Lx^2 \right]_0^L = -\left[ \frac{1}{4}L^4 + \frac{1}{2}L^2 - \frac{2}{3}L \cdot L^2 \right]_0^L \]

\[ = - \left[ \frac{L^4}{4} + \frac{L^4}{2} - \frac{2}{3}L^4 \right] = -\left( \frac{3 + 6 - 8}{12} \right) L^4 \]

\[ = -\frac{1}{12}L^4 \]

\[ X_c = \frac{-\frac{1}{12}L^4}{-\frac{L^3}{3}} = \frac{1}{4}L \]

b)

\[ \Sigma F_x = 0 \rightarrow A_x = 0 \]

\[ \Sigma F_y = 0 \rightarrow A_y = R = \frac{L^3}{3} \]

\[ \Sigma M_A = 0 \rightarrow M_A - R (\frac{L}{4}) = 0 \]

\[ M_A = (\frac{L^3}{3})(\frac{L}{4}) \rightarrow M_A = \frac{L^4}{12} \]

\[ \overrightarrow{A} = \frac{L^3}{3} \]

\[ \overrightarrow{R} = \frac{L^3}{3} \]
Problem #4 (25%)  

The truss shown below is supported by two smooth pins at E and F. Using the Method of Joints, determine the force in members AB, AJ, BJ, BC and BH and state whether each member is in tension (t), or compression (c).

![Truss Diagram]

**Note:** You need to draw all relevant FBD’s and show calculation details to receive credit.

\[
\begin{align*}
\text{FBD of J A:} \\
\Sigma F_y &= 0 \\
F_{AJ} \left( \frac{1}{\sqrt{2}} \right) - 50 &= 0 \\
F_{AJ} &= 50 \sqrt{2} \text{ kN (t)} = 70.7 \text{ kN (t)} \\
\Sigma F_x &= 0 \\
50 \sqrt{2} \left( \frac{1}{\sqrt{2}} \right) + F_{AB} &= 0 \\
F_{AB} &= -50 \text{ kN} = 50 \text{ kN (c)}
\end{align*}
\]

**FBD of J J**

\[
\begin{align*}
\Sigma F_y &= 0 \\
-50 \sqrt{2} \left( \frac{1}{\sqrt{2}} \right) - F_{BJ} &= 0 \\
F_{BJ} &= 50 \text{ kN} = 50 \text{ kN (c)}
\end{align*}
\]

**FBD of J B**

\[
\begin{align*}
\Sigma F_y &= 0 \\
-50 + F_{BH} \left( \frac{1}{\sqrt{2}} \right) &= 0 \\
F_{BH} &= 50 \sqrt{2} \text{ kN} = 70.7 \text{ kN (c)}
\end{align*}
\]

\[
\begin{align*}
\Sigma F_x &= 0 \\
50 \sqrt{2} \left( \frac{1}{\sqrt{2}} \right) - 50 + F_{BC} &= 0 \\
F_{BC} &= -100 \text{ kN} = 100 \text{ kN (c)}
\end{align*}
\]

\[
\begin{align*}
F_{AB} &= 50 \text{ kN (c)} \\
F_{AJ} &= 70.7 \text{ kN (t)} \\
F_{BJ} &= 50 \text{ kN (c)} \\
F_{BC} &= 100 \text{ kN (c)} \\
F_{BH} &= 70.7 \text{ kN (t)}
\end{align*}
\]