RENSSELAER POLYTECHNIC INSTITUTE
TROY, NY
EXAM NO. 3 INTRODUCTION TO ENGINEERING ANALYSIS
(ENGR-1100) – Fall 12

NAME: Solution  Section: __________

RIN: __________________________

Wednesday, November 14, 2012
8:00 – 9:50

Please state clearly all assumptions made in order for full credit to be given.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Good Luck!
Problem #1 (25%)

\[ 2x_1 + x_2 + 4x_3 = -2 \]
\[ 4x_1 - x_2 + 3x_3 = -7 \]
\[ 3x_1 - 2x_2 - x_3 = -3 \]

a) Express the three equations in the form AX = B \hspace{1cm} (3)
b) Using the method learned in class determine the inverse of matrix A \hspace{1cm} (12)
c) Use the inverse found in b) to solve for \( x_1, x_2, \) and \( x_3 \). \hspace{1cm} (6)
d) Using the method of cofactor expansion, determine the determinant of matrix A by expanding along the second column \hspace{1cm} (4)

\textbf{Note:} You need to show all intermediate work to receive credit. Answers not in the box will not be graded.

\[ \begin{bmatrix}
1 & -\frac{1}{2} & \frac{4}{7} \\
0 & \frac{8}{7} & -1 \\
0 & 0 & \frac{5}{7}
\end{bmatrix} \]
\[ \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
\frac{a}{14} & -\frac{1}{2} & \frac{4}{7} \\
\frac{8}{7} & -1 & \frac{4}{7} \\
-\frac{5}{7} & \frac{1}{7} & -3/7
\end{bmatrix}^{-1} \begin{bmatrix}
-2 \\
-3 \\
-3
\end{bmatrix}
\]
\[ x_1 = -\frac{3}{4} + \frac{7}{2} - \frac{12}{7} = \frac{1}{2} \]
\[ x_2 = -\frac{14}{4} + 7 - \frac{12}{7} = 3 \]
\[ x_3 = \frac{5}{4} - \frac{7}{2} + \frac{9}{7} = \frac{3}{2} \]

\[ \det(A) = 14 \]

\[ \det(A) = -1 \left[ (-4) - (12) \right] - 1 \left[ (-1) - (12) \right] + 2 \left[ (8) - (16) \right] = 16 + 14 - 16 = 14 \]
Problem #2 (25%)  

The bent beam shown here is attached to the wall through the cable $BC$ where $C$ is a point in $yz$ plane. At $A$ there is a smooth square rod which fits loosely through the square hole of the collar. This support restricts the motion in both the $x$ and the $y$ direction and also restrains the rotation in all three directions. The force $F$ is 45 lb.  

a) Draw the free-body diagram of the bent beam.

b) Determine tension in cable $BC$ and support reactions (forces and moments) at $A$ as vectors in Cartesian coordinates.

<table>
<thead>
<tr>
<th>Tension vector</th>
<th>$\vec{T} = (-89.895i + 30j + 45k)$ lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support reaction force vector at $A$</td>
<td>$89.895i - 30j$ lb</td>
</tr>
<tr>
<td>Support reaction moment vector at $A$</td>
<td>$-180i - 719.58k$ lb ft</td>
</tr>
</tbody>
</table>

$$\vec{T} = \vec{T} \cdot \vec{\lambda}_{BC} = \vec{T} \cdot \frac{-12i + 4j + 6k}{\sqrt{144 + 16 + 36}} = \vec{T} \cdot \frac{-12i + 4j + 6k}{14}$$

$$\vec{T} = \vec{T} (-0.857i + 0.286j + 0.429k)$$

$$\sum F_x = 0 \rightarrow -0.857T + A_x = 0 \rightarrow A_x = 89.895 \text{ lb}$$

$$\sum F_y = 0 \rightarrow -0.286T + A_y = 0 \rightarrow A_y = -30 \text{ lb}$$

$$\sum M_A = \vec{0} \rightarrow M_{Ax}i + M_{Ay}j + \vec{r}_{AB} \times \vec{T} + \vec{r}_{AD} \times \vec{F} = \vec{0}$$

$$M_{Ax}i + M_{Ay}j + M_Ak + \begin{bmatrix} i & j & k \\ 12 & 40 & 0 \\ -89.895 & 30 & 45 \end{bmatrix} + \begin{bmatrix} i & j & k \\ 12 & 0 & 0 \\ 0 & 0 & -45 \end{bmatrix} = \vec{0}$$

$$M_{Ax}i + M_{Ay}j + M_Ak + 180i - 540j + 719.58k + 540j = \vec{0}$$
\((M_{Ax} + 180)\bar{c} + (M_{Ay})\bar{f} + (M_{Az} + 719.58)\bar{x} = 0\)

\[M_{Ax} = -180 \text{ lb. ft}\]
\[M_{Ay} = 0\]
\[M_{Az} = -719.58 \text{ lb. ft}\]
Problem #3 (25%)

Wing AB of an aircraft can be modeled as a beam with length L welded at A to the fuselage as shown in the figure. The point of contact A between the wing and the fuselage may be considered as a fixed support. The lift force is approximated as a distributed load with the distribution function \( w = a - bx^2 \text{ lb/ft} \) along AB. Moreover, the load due to the wing structure weight is approximated as a distributed load with the uniform distribution \( \rho \text{ lb/ft} \) as shown in the figure, where \( a, b, \) and \( \rho \) are positive numbers.

a) Find the resultant lift force vector (in the Cartesian coordinates) and the associated point of application \( (x_{c1}) \) measured from A as a function of \( a, b, \) and \( L \).

<table>
<thead>
<tr>
<th>Resultant lift force vector</th>
<th>( R_1 = (aL - \frac{1}{3}bl^3) \text{ lb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{c1} )</td>
<td>( \frac{1}{2}aL - \frac{1}{4}bl^3 )</td>
</tr>
<tr>
<td></td>
<td>( a - \frac{1}{3}bl^2 )</td>
</tr>
</tbody>
</table>

b) Find the weight vector of entire wing (in the Cartesian coordinates) and the associated point of application \( (x_{c2}) \) as a function of \( \rho, \) and \( L \).

<table>
<thead>
<tr>
<th>Weight vector</th>
<th>( R_2 = -\rho L \text{ lb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{c2} )</td>
<td>( x_{c2} = L/2 )</td>
</tr>
</tbody>
</table>

(c) Find and express the reaction forces and moment at A as vectors in the Cartesian coordinates as functions of \( a, b, \rho, \) and \( L \).

<table>
<thead>
<tr>
<th>Support reaction force vector at A</th>
<th>( R_A = \left[ -\left(aL - \frac{1}{3}bl^3\right) + \rho L \right] \text{ lb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support reaction moment vector at A</td>
<td>( -\left(\frac{1}{2}aL^2 - \frac{1}{4}bL^4\right) + \rho \frac{L^2}{2} \text{ lb-ft} )</td>
</tr>
</tbody>
</table>

\[ w = a - bx^2 \text{ lb/ft} \]

\[ R_1 = \int_0^L (a - bx^2)dx = \left[ ax - \frac{1}{3}bx^3 \right]_0^L = aL - \frac{1}{3}bl^3 \text{ lb} \]
\[ x_{c1} = \frac{\int_0^L (a-bx^2) \, dx}{R_2} = \frac{\int_0^L (ax-bx^2) \, dx}{aL - \frac{1}{3}bl^3} = \frac{\left[ \frac{1}{2}ax^2 - \frac{1}{3}bx^3 \right]_0^L}{aL - \frac{1}{3}bl^3} = \frac{\frac{1}{2}al^2 - \frac{1}{4}bl^4}{aL - \frac{1}{3}bl^3} = \frac{\frac{1}{2}al - \frac{1}{4}bl^3}{aL - \frac{1}{3}bl^3} \]

\[ M_{Az} + \left( al - \frac{1}{3}bl^3 \right) \left( \frac{1}{2}al^2 - \frac{1}{4}bl^4 \right) - \rho L \frac{L}{2} = 0 \]

\[ M_{Az} = -\left( \frac{1}{2}al^2 - \frac{1}{4}bl^4 \right) + \rho \frac{L^2}{2} \text{ lb ft} \]

\[ \sum F_x = 0 \rightarrow A_x = 0 \]

\[ \sum F_y = 0 \rightarrow A_y + R_1 - R_2 = 0 \]

\[ A_y + aL - \frac{1}{3}bl^3 - \rho L = 0 \]

\[ \sum M_A = 0 \rightarrow M_{Az} + R_1 x_{c1} - R_2 x_{c2} = 0 \]

\[ R_2 = \rho L \text{ lb} \]
Problem #4 (25%) 

The truss shown below is supported by smooth pins at D and E. 

a) Determine the reaction components at each support. 

b) Determine the force in members $AB$, $AG$, $CG$, $FG$ and $CF$ and state whether each member is in tension (t), or compression (c). 

Note: You need to draw all relevant FBD’s and show calculation details to receive credit.
FBD of pin C:

\[ \sum F_y = 0 \]
\[ -CF - \frac{2.25}{\sqrt{3}} = 0 \] ①

\[ CF = -1 \text{ kN} = 1 \text{ kN} (\downarrow) \] ①