Summary of the theory (Sec. 3.2)

$$L[y] = y'' + p(x)\,y' + q(x)\,y = 0.$$

p, q continuous on $I = (a, b)$.

1. Given initial data $y(x_0) = y_0$, $y'(x_0) = y_0'$, there exists exactly one solution throughout I.

2. If y_1, y_2 are solutions $\Rightarrow$ $c_1 y_1 + c_2 y_2$ is a solution

3. Two solutions y_1 and y_2 form a <u>fundamental set</u> if any solution y is their combination

$$y(x) = c_1 y_1 + c_2 y_2$$

a) $\{y_1, y_2\}$ is a fundamental set <u>iff</u> (means if and only if) $W(\tilde{x}_0) \neq 0$ at some $\tilde{x}_0$.

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$$

b)* If $W(\tilde{x}_0) \neq 0$ then $W(x) \neq 0$ for any $x \in I$.

c) Always, there is a fundamental set.
(There are infinitely many sets of fund. solutions)

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*) A proof is explained in Thm 3.3.2