MATH-2400 Fall '04 **Exam #1** **Name:**______________________________ **Sec.**

Please answer all **5** questions, showing your work in reasonable detail. You are allowed one sheet of notes. Other notes books, laptops and **calculators** are not permitted.

1. (a)(10 pts) Solve the initial value problem:

$$t^2 \frac{dy}{dt} + 2ty = \sin t, \quad y(\pi) = 1.$$

What is the range of t for which the solution exists?
(b) (3 pts BONUS) Find the initial condition y_0 at $t_0 = \pi$, $y(\pi) = y_0$ so that $y(t)$ would exist for all t.

2.(a) (6p.) Solve **explicitly** the initial value problem $y' = -t/y; \quad y(0) = 3$, and find the interval where the solution is valid.

2.(b) (4p.) Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Set up an initial value problem for the temperature of a cup of coffee in a room at 70°F, if it has a temperature of 200°F when freshly poured. **Do not solve the problem**. Comment on whether all the data are provided that are necessary for setting the problem up.

3.(a) (8p.) Solve the initial value problem: $y'' + 6y' + 9y = 0,\ y(0) = 1,\ y'(0) = 2.$

3.(b) (2p.) Let $\phi(t)$ be a solution of the differential equation $y'' + y' + q(t)y = g(t)$, where $g(t)$ is not zero. Will $y = c\phi(t)$ be also a solution? Explain your answer.

4. (10p.) (a) Find the real general solution of the equation $y'' + 4y' + 5y = 0$.
(b) Describe behavior of solutions for increasing t.
(c) Use Euler's formula to write the following expressions in the form $a + ib$: $\exp(i\pi)$, 3^{2-i}, $\exp(2 + 2i)$.
(d) (Bonus 2p.) Write $2i$ in the form $re^{i\theta}$.

5(a).(6p.) For the equation $y' = 2y - y^2$ determine the critical (equilibrium) points and classify each one as asymptotically stable or unstable. Draw the phase line, and in the ty-plane sketch graphs of **five** solutions, with $y_0 = -1$, 0, 1, 2, 3.
(b) (4p.) For the equation $y' = -y + 1$ sketch a direction field (say, on the rectangle $0 \le t \le 5, -1 \le y \le 3$).
(c) Based on the direction field, determine the behavior of y as $t \to \infty$.
(d) (Bonus 3p.) To confirm the behavior, draw the phase line and find the general solution of the equation.