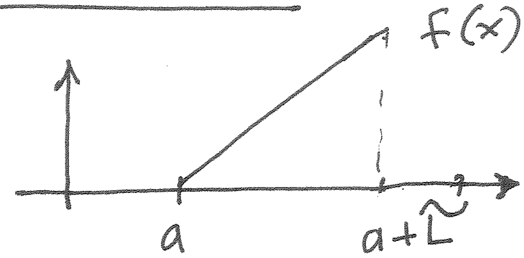
Primer of Fourier series

Given a function



It can be expanded into many different kinds of Fourier series.

We need only full Fourier series (mostly for introductory explanations), and sine and cosine Fourier series (for solving boundary - initial value problems).

$$f(x) = \frac{\tilde{a}_0}{2} + \sum_{n=1}^{\infty} \left(\tilde{a}_n \cos \frac{n\pi x}{\tilde{L}/2} + \tilde{b}_n \sin \frac{n\pi x}{\tilde{L}/2} \right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{\tilde{L}}$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{\tilde{L}}$$

Note different frequencies $\frac{2n\pi}{\tilde{L}}$, vs. $\frac{n\pi}{\tilde{L}}$!

The coefficients are given by <u>Euler–Fourier formulas:</u>

$$\tilde{a}_n = \frac{1}{\tilde{L}/2} \int_a^{a+\tilde{L}} f(x) \cos \frac{n\pi(x-a)}{\tilde{L}/2} dx$$

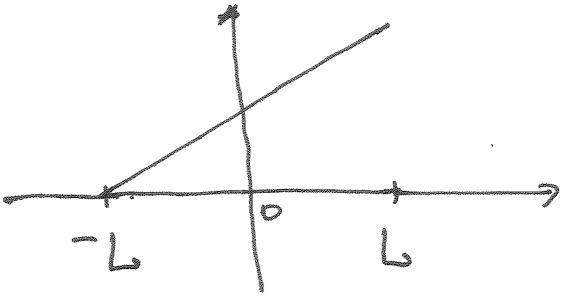
$$\tilde{b}_n = \frac{1}{\tilde{L}/2} \int_a^{a+\tilde{L}} f(x) \sin \frac{n\pi(x-a)}{\tilde{L}/2} dx$$

$$a_n = \frac{2}{\tilde{L}} \int_a^{a+\tilde{L}} f(x) \cos \frac{n\pi(x-a)}{\tilde{L}} dx$$

$$b_n = \frac{2}{\tilde{L}} \int_a^{a+\tilde{L}} f(x) \sin \frac{n\pi(x-a)}{\tilde{L}} dx$$

Note that usually $a = 0$ (see next page)

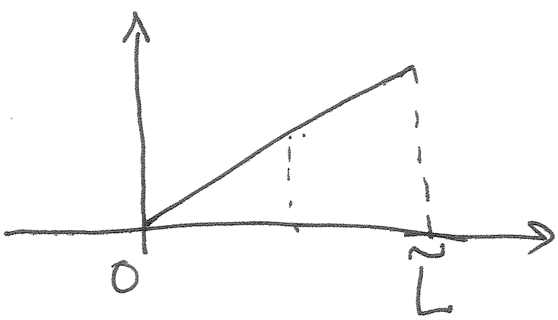
"Centered" version of Fourier series.



$$f(x) = \frac{\tilde{a}_0}{2} + \sum_{n=1}^{\infty}\left(\tilde{a}_n \cos\frac{n\pi x}{L} + \tilde{b}_n \sin\frac{n\pi x}{L}\right)$$

$$\tilde{a}_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx; \quad \tilde{b}_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

or



$$(c) \quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{\tilde{L}}$$

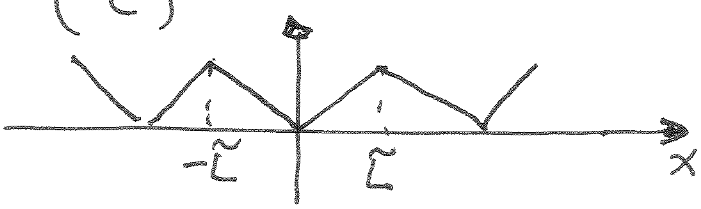
$$a_n = \frac{2}{\tilde{L}}\int_0^{\tilde{L}} f(x)\cos\frac{n\pi x}{\tilde{L}}\,dx$$

note $\tilde{L} = 2L$

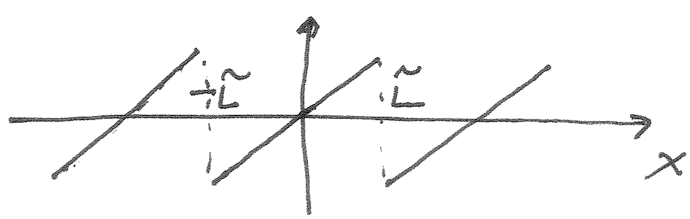
$$(s) \quad f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{\tilde{L}}$$

$$b_n = \frac{2}{\tilde{L}}\int_0^{\tilde{L}} f(x)\sin\frac{n\pi x}{\tilde{L}}\,dx$$

(c)



(s)



(full series)