Math-6500 Partial Differential Equations Fall 2007
Assignment 6

Due Thursday, December 12, by 4pm.

Integral Equations

1. Consider the integral equation

$$u(x) = f(x) + \lambda \int_0^{10} xtu(t)dt$$

   (a) For which λ does the equation have a unique solution for any f?

   (b) For which λ does the equation have no solution for a generic f and infinitely many solutions for a "special" admissible f ? Describe the admissible functions f and solve the equation.

2. (Problem 7-4 #9(a-d) c*). A* Volterra integral operator K is defined on $C[a, b]$ as

$$(Ky)(x) = \int_a^x k(x, s)y(s)ds.$$

   where $k(x, s)$ is assumed to be continuous on the triangle $a \leq s \leq x \leq b$. Show that the integral equation $\phi + \lambda K\phi = f$ has a unique solution for each $f \in C[a, b]$ and any λ. Hint: GL give detailed instructions how to solve this problem.

3. Solve the integral equations

   (a)
$$y(t) = t + \int_0^1 (1 + st)y(s)ds$$

   (b)
$$y(t) = f(t) + \lambda \int_0^{2\pi} \sin s \sin t \; y(s)ds$$

4. Problems 7.2.3 and 7.8.7 from *GL*.