

Please answer all 5 questions, showing your work in reasonable detail. You are allowed one sheet of notes. Other notes books, laptops and **calculators** are not permitted.

1. (a)(10 pts) Solve the initial value problem:

$$t^2 \frac{dy}{dt} + 2ty = \sin t, \quad y(\pi) = 1.$$

What is the range of t for which the solution exists?

(b) (3 pts BONUS) Find the initial condition y_0 at $t_0 = \pi$, $y(\pi) = y_0$ so that $y(t)$ would exist for all t .

2.(a) (6p.) Solve **explicitly** the initial value problem $y' = -t/y$; $y(0) = 3$, and find the interval where the solution is valid.

2.(b) (4p.) Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Set up an initial value problem for the temperature of a cup of coffee in a room at 70°F , if it has a temperature of 200°F when freshly poured. **Do not solve the problem.** Comment on whether all the data are provided that are necessary for setting the problem up.

3.(a) (8p.) Solve the initial value problem: $y'' + 6y' + 9y = 0$, $y(0) = 1$, $y'(0) = 2$.

3.(b) (2p.) Let $\phi(t)$ be a solution of the differential equation $y'' + y' + q(t)y = g(t)$, where $g(t)$ is not zero. Will $y = c\phi(t)$ be also a solution? Explain your answer.

4. (10p.) (a) Find the real general solution of the equation $y'' + 4y' + 5y = 0$.

(b) Describe behavior of solutions for increasing t .

(c) Use Euler's formula to write the following expressions in the form $a + ib$: $\exp(i\pi)$, 3^{2-i} , $\exp(2 + 2i)$.

(d) (Bonus 2p.) Write $2i$ in the form $re^{i\theta}$.

5(a).(6p.) For the equation $y' = 2y - y^2$ determine the critical (equilibrium) points and classify each one as asymptotically stable or unstable. Draw the phase line, and in the ty -plane sketch graphs of **five** solutions, with $y_0 = -1, 0, 1, 2, 3$.

(b) (4p.) For the equation $y' = -y + 1$ sketch a direction field (say, on the rectangle $0 \leq t \leq 5, -1 \leq y \leq 3$).

(c) Based on the direction field, determine the behavior of y as $t \rightarrow \infty$.

(d) (Bonus 3p.) To confirm the behavior, draw the phase line and find the general solution of the equation.