

### Exam 3. Answers.

1. a)  $u(x,t) = 3 \sin 2x e^{-20t}$

b)  $(T(t) \sin 2x)_t + T(t) \sin 2x = (T(t) \sin 2x)_{xx} \cdot 5$   
 $T(0) \sin 2x = 3 \sin 2x \Rightarrow T' + T = -20T, T(0) = 3$   
 $T(t) = 3 e^{-21t}$

2.  $y_n = \sin\left(\frac{\pi}{6} + \frac{n\pi}{3}\right)x = \sin \frac{2n+1}{6} \pi x, \lambda_n = \left[\frac{2n+1}{6} \pi\right]^2$   
 $n = 0, 1, 2, \dots$

3. 2b)  $X''Y' + X''Y + XY = 0$

$$X''(Y' + Y) + XY = 0 \Rightarrow \frac{X''}{X} = -\frac{Y}{Y'+Y} = \lambda$$

$$X'' - \lambda X = 0 \quad \lambda Y' + \lambda Y + 1 = 0$$



$$C_0 = \frac{4}{3} \quad C_1 = \frac{2}{\pi} \sin \frac{\pi x}{3} \Big|_1^3 = -\frac{\sqrt{3}}{\pi}$$

4.  $\begin{bmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{bmatrix}_{(0,0)} = \begin{bmatrix} \sin y & (1+x) \cos y \\ -1 & \sin y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

e-values  $\pm i \Rightarrow$  center  $\Rightarrow$  no stability info.

5. a)  $f \equiv 0$  b)  $v(x) = \frac{10}{L+1} (x+1)$