Math-2400 Summer 2002
Sample Problems for Test #2: Answers

1. Determine a suitable form for $Y(t)$ if the method of undetermined coefficients is to be used to solve the following differential equation:

$$y'' + y = e^t \sin t + \cos t + t^2$$

DO NOT evaluate the coefficients

$$\boxed{\text{Y=Ae}^t \text{sint+Be}^t \text{cost+t(Csint+Dcost)+Et}^2\text{+Ft+G}}$$

2. (a) Some damped vibrating system is described by the equation

$$u'' + 4u' + 5u = 0.$$

If the initial position is equal to 1, and the initial velocity is 0, describe the motion of the system for all time. Express the solution in the form $u = Ae^{-\beta t}\cos(\mu t - \delta)$ for some suitable constants A, β, μ.

2.(b) Consider the forced equation,

$$u'' + \gamma u' + 4u = 2\sin t,\ \gamma = 1,$$

then one part of its solution is decaying with time (a transient part) while another part stays of the same magnitude and dominates the solution for large time (steady state). Calculate the steady solution.

2. (c) For the equation , $u'' + 4u = 2\sin \omega t$, for which value of ω will the mechanical system experience a resonance?

(a)

$$u = e^{-2t}(A\cos t + B\sin t) = e^{-2t}(\cos t + 2\sin t) = \sqrt{3}e^{-2t}\cos(t - \delta)$$

where $\delta = \tan^{-1} 2$. (b) First, $u = Y + u_{\text{hom}}$, where $u_{\text{hom}} = e^{-\gamma/2}(A\cos\frac{\sqrt{15}}{2}t + B\sin\frac{\sqrt{15}}{2}t)$ which decays exponentially with time.

$$Y = C\sin t + D\cos t = \frac{3}{5}\sin t - \frac{1}{5}\cos t$$

This is the steady solution. (c) $\omega = 2$.

3. Solve the problem

$$y'' + 4y' + 4y = 0,\ y(0) = -1,\ y'(0) = 1$$

$\boxed{y = -e^{-2t} - te^{-2t}}$

4. Find the general real solution of the problem $y'' + 4y' + 5y = 0$. $\boxed{y = e^{-2t}(C\sin t + D\cos t)}$

5. Find the general solution of the problem $y'' + 4y = t^2$. $\boxed{y = C\sin 2t + D\cos 2t + t^2/4 - 1/8}$

6. Write the following numbers in the form $a + ib$: $e^{i\pi/4}$, $5^{i/2}$; write i, 1 in the form $e^{i\theta}$. $\boxed{\frac{\sqrt{2}}{2}(1+i),\ \cos\frac{\ln 5}{2} + i\sin\frac{\ln 5}{2},\ \ i = e^{i\pi/2},\ 1 = e^{i0}}$

7. Find the Wronskian of e^t and te^t. $\boxed{W[e^t, te^t] = e^{2t}}$

8. Use the method of variation of parameters to find a particular solution of the equation $y'' + 4y' + 4y = e^{-2t}/t^2$.

$\boxed{Y = -e^{-2t}\ln|t|}$