Mathematical Analysis I. Math-4200, Fall 2006
Assignment 9

Due Thursday, November 16 (Either in class, or my mailbox in AE 301, or under my door AE 405).

Reading:

Nov. 6 and 9: Sections 6.1-6.2.
Nov. 13 and 16: Sections 6.3-7.2.

Problems

You are welcome to consult the text and notes and discuss the problems with other people. However, the solutions should be *yours*. Please indicated on your papers, who you discussed the problems with.

1. Give an example of a Riemann integrable function f such that the derivative of $F(x) = \int_0^x f(t)\,dt$ does not exist at some point x_0. (Compare to Problem 6.2.4 #10)

2. Prove the integral mean value theorem (6.1.5 #4). Hint: The theorem can be stated as the regular mean value theorem (p. 160) for some function, which one?

3. Problem 6.1.5 #10. Linearity means $A_w(\lambda f + \mu g) = \lambda A_w(f) + \mu A_w(g)$.

4. If f is a Riemann integrable function on [0,1] then prove that $F(x) = \int_0^x f(t)\,dt$ is continuous (Problem 6.2.4 9a). Hint: In other words, you need to show that

$$|F(x) - F(x_0)| = |\int_{x_0}^{x} f(t)\,dt|$$

is small if $|x - x_0|$ is small.

5. For which values of a and b does the improper integral

$$\int_0^{1/2} x^a |\ln x|^b dx$$

converge? Hint: Consider the cases $a > -1, a = -1, a > -1$.