Mathematical Analysis I. Math-4200, Fall 2006
Assignment 8

Due Thursday, November 9 (Either in class, or my mailbox in AE 301, or under my door AE 405).

Reading:

Oct. 23 and Nov. 2: Sections 5.2-5.4.
Nov. 6 and 9: Sections 6.1-6.2.

Problems

You are welcome to consult the text and notes and discuss the problems with other people. However, the solutions should be *yours*. Please indicated on your papers, who you discussed the problems with.

1. Problem 5.1.3 #9: A "zoom" on the graph of $y = f(x)$ near (x_0, y_0) (with $y_0 = f(x_0)$) with magnification factor M is the graph of the function defined by $f(x_0 + x/M) = y_0 + y/M$. Prove that if f is differentiable at x_0, then the zoom converges to the straight line through the origin with the slope $f'(x_0)$, as $M \to \infty$. What happens to the zoom of $|x|$ near the origin. **Remark:** This problem is mostly a reading exercise. The main thing is to understand how a new function $x \to \varphi(x)$ is defined by the equation in the statement of the problem. It might help to do the construction for some simple function, such as x^2 at $x_0 = 1$.

2. Problem 5.2.3 #2.

3. Problem 5.3.4 #7. Prove that $.f(x) = x^{1/k}$ can be defined on $[0, \infty)$ by the requirement that it be the inverse function of $g(x) = x^k$ on $[0, \infty)$, where k is any positive integer. Use the inverse function theorem to derive the usual formula for f'. **Remark:** The proof part here will consist in reading the inverse function theorem (even not necessarily its proof) and checking that the theorem applies in the case of x^k.

4. Problem 5.3.4 #13. **Hint:** Use the chain rule.

5. Problem 5.2.4 #3. **Remark:** This problems *is* extremely easy.

6. Another easy problem: Problem 5.4.6 #20. Use the second order Taylor theorem with *Lagrange* remainder (not Cauchy as I mistakenly stated in class) to estimate $101^{1/2}$.