

Mathematical Analysis I. Math-4200, Fall 2006
Assignment 7

Due Thursday, October 26 (Either in class, or my mailbox in AE 301, or under my door AE 405).

Reading:

Oct. 16 and 19: Sections 4.2–5.1.1

Oct. 23 and Nov. 2: Sections 5.1-5.3.

Problems

You are welcome to consult the text and notes and discuss the problems with other people. However, the solutions should be *yours*. Please indicated on your papers, who you discussed the problems with.

1. (Problem 4.2.4 #7) Let f be a monotone function on an interval. Prove that if the image of f is an interval, then f is continuous. [Hint: Prove by contradiction, use results of Thm. 4.2.6.] Give an example of a nonmonotone function on an interval whose image is an interval but that is not continuous.
2. (a) Give an example of two functions f and g that are uniformly continuous while their product fg is not uniformly continuous. You will have to make them unbounded (see Prob. 4.2.4 #10).
(b) Give an example of a function on \mathbb{R} that assumes its sup and inf on any compact interval and yet is not continuous.
3. Problem 4.2.4 #11.

Calculus of O and o

4. (Problems 5.1.3 #1 & 3) Show that $f(x) = O(|x - x_0|^2)$ as $x \rightarrow x_0$ implies $f(x) = o(|x - x_0|)$ as $x \rightarrow x_0$, but give an example to show that the converse is not true. Show that $f(x) = O(|x - x_0|^k)$ and $g(x) = o(|x - x_0|^j)$ imply $fg(x) = o(|x - x_0|^{k+j})$
5. Problems 5.1.3 #2 &4.
6. Problem 5.1.3 #5.

E8. (Extra credit). Problem 4.5.4 #17: Give an example of a function on \mathbb{R} that has the intermediate value property for every interval (it takes on all values between $f(a)$ and $f(b)$ on $a \leq x \leq b$) but fails to be continuous at a point. Can such a function have jump discontinuities?