**Mathematical Analysis I. Math-4200, Fall 2006**
**Assignment 10**

**Due Thursday, November 30 (**Either in class, or my mailbox in AE 301, or under my door AE 405).

## Reading:

Nov. 13, 16 and 20: Sections 7.1-7.2.
Nov. 27: Section 7.3.

## Problems

You are welcome to consult the text and notes and discuss the problems with other people. However, the solutions should be *yours.* Please indicated on your papers, who you discussed the problems with.

1. If $f(x)$ is a continuous complex-valued function and $f(x) \neq 0$ for any $x$ in the domain, prove that $1/f(x)$ is continuous (Problem 7.1.3 #3).

2. (Problem 7.1.3 #10).

   (a) Use the triangle inequality to prove that
   $$||z| - |z_1|| \leq |z - z_1|$$

   (b) If $z = x + iy$, $z_1 = x_1 + iy_1$, prove $||z| - |z_1|| \leq |x - x_1| + |y - y_1|$.

3. (Problem 7.2.4 #1). Give an example of two convergent series $\sum_{k=1}^{\infty} x_k$ and $\sum_{k=1}^{\infty} y_k$ such that $\sum_{k=1}^{\infty} x_k y_k$ diverges. Prove that this cannot happen if one of the series converges absolutely.

4. (Problem 7.2.4 #3). Show that it is not true that for every error $1/m$ there exists $n$ such that $\left|\sum_{k=1}^{n} r^k - r/(1-r)\right| < 1/m$ for all $r$ in $0 < r < 1$.

5. Prove the root test (Theorem 7.2.3b).

**E9. (Extra credit).** Show that if $\sum_{k=1}^{\infty} a_k$ is absolutely convergent, there exists an absolutely convergent series $\sum_{k=1}^{\infty} b_k$ such that $\lim_{k\to\infty} a_k/b_k = 0$. Explain why this result shows that there is no "universal" comparison series for testing absolute convergence.