Linear Algebra. Math-4100, Fall 2007
Assignment 5

Due Thursday, October 18, by 4pm. (Either in class, or my mailbox in AE 301, or under my door AE 405).

Reading

Oct. 8, 11: **Strang** Sections 5.1, 5.2.

Oct. 15 and 18: **Strang** Sections 5.1–5.3; 7.1; **Gelfand** Section 9.

Problems

You are encouraged to consult the text and notes and discuss the problems with other people. However, the solutions should be *yours*. Please indicated on your papers, who you discussed the problems with. *Please submit extra credit problems on a separate sheet of paper.*

1. (a) Problem 5.1 #16.
   (b) A skew-symmetric matrix has $K^T = -K$. Use properties of determinants, in particular linearity and $\det A = \det A^T$, to prove that all skew symmetric matrices in odd dimensions are singular.
2. Problem 5.1 #24.
3. Problem 5.1 #25.
4. Problem 5.1 #28.
5. Problem 5.2 #6: Place the smallest number of zeros in a 4×4 matrix that will guarantee $\det A = 0$ no matter how the nonzero entries are selected. Place as many zeros as possible while still allowing $\det A \neq 0$.
6. Problem 5.2 #11: How many permutations of $(1, 2, 3, 4)$ are even and what are they? A permutation is called *even* if it can be obtained from $(1, 2, 3, 4)$ with an even number of exchanges. How many perturbations of $(1, 2, ..., n)$ are even?
7. Problem 5.2 #14.
8. Problem 5.2 #25. Note that the key idea for designing counterexamples is contained in Problem 27.
9. Problem 5.3 #1(b).
10. Problem 5.3 #21.

E5. (Extra credit). Calculate the n-th order determinant

$$\Delta = \begin{vmatrix} x & a & a & \dots & a \\ a & x & a & \dots & a \\ a & a & x & \dots & a \\ . & . & . & \dots & . \\ a & a & a & \dots & x \end{vmatrix}$$

E6. (Based on Problem 5.3 #11 for professors only). If you know all n^2 cofactors of a $n \times n$ invertible matrix A, how would you find A?