

Photometric Separation of Stellar Properties Using SDSS Filters

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ABSTRACT

Using synthetic photometry of Kurucz model spectra, we explore the colors of stars as a function of temperature, metallicity, and surface gravity with Sloan Digital Sky Survey (SDSS) filters, $u'g'r'i'z'$. The synthetic colors show qualitative agreement with the few published observations in these filters. We find that the locus of synthetic stars is basically two-dimensional for $4500 \text{ K} < T < 8000 \text{ K}$, which precludes simultaneous color separation of the three basic stellar characteristics we consider. Colors including u' contain the most information about normal stellar properties; measurements in this filter are also important for selecting white dwarfs. We identify two different subsets of the locus in which the loci separate by either metallicity or surface gravity. For $0.5 < g' - r' < 0.8$ (corresponding roughly to G stars), the locus separates by metallicity; for photometric error of a few percent, we estimate metallicity to within ~ 0.5 dex in this range. In the range $-0.15 < g' - r' < 0.00$ (corresponding roughly to A stars), the locus shows separation by surface gravity. In both cases, we show that it is advantageous to use more than two colors when determining stellar properties by color. Strategic observations in SDSS filters are required to resolve the source of a $\sim 5\%$ discrepancy between synthetic colors of Gunn-Stryker stars, Kurucz models, and external determinations of the metallicities and surface gravities. The synthetic star colors can be used to investigate the properties of any normal star and to construct analytic expressions for the photometric prediction of stellar properties in special cases.

1. Introduction

The Sloan Digital Sky Survey (SDSS) is designed to obtain high precision photometry and spectroscopy for one-fourth of the sky to a limiting magnitude of $V \approx 20$ (the galaxy redshift sample will be somewhat brighter than this), with a goal of mapping the Universe in redshift (Gunn & Knapp 1993; Kent 1994; Gunn 1995). A survey of such depth and coverage will produce data for unprecedented numbers of a variety of objects; in particular, there will be high-quality photometric data for perhaps 10^8 stars and 10^6 quasar candidates (Gunn & Knapp 1993; Kent 1994).

The photometric system for the survey uses five broad-band filters, u' , g' , r' , i' , and z' , covering 3000–11500 Å (Fig. 1). The SDSS will use the photometric data to select galaxy and quasar candidates and will then obtain spectra for those candidates. The SDSS photometric system differs from previous standard systems, as outlined in §2.

Since the SDSS is expected to produce a large amount of data on a nonstandard photometric system, it is desirable to characterize the system in advance of the actual survey. For example, synthetic photometry can be used to gain insight into the properties of the photometric system, to estimate how the colors of objects are affected by variations in the intrinsic attributes of the objects and how those variations compare with expected photometric errors, and to estimate the regions of color space different types of objects occupy. The last issue is particularly important for the SDSS because one would like to characterize the overlap in color space of, e.g., the stellar and quasar loci, to maximize the number of quasar spectra obtained. Richards et al. (1997, hereafter R97) discuss some of the overlap observed in their photometry, which uses SDSS filters. Synthetic photometry plays a key role in identifying potentially interesting or unusual objects, and is vital to the efficient analysis of the amount of data the SDSS will produce.

We will apply the notation $u'g'r'i'z'$ to our calculated colors, with the caveat that the

calibration of the SDSS system is in progress and the comparison of our calculations to SDSS data in principle requires transformation of our colors into the final calibrated SDSS system colors (in any case, no simulation can exactly reproduce or replace observation). These simulations mimic the final SDSS system to the extent that the description in Fukugita et al. (1996, hereafter F96) reflects the actual characteristics of the system.

The purpose of this paper is twofold. We first present synthetic colors on the SDSS system for several sets of stellar spectra; this synthetic photometry can aid in the identifications of objects found in the SDSS photometric survey. Then, we show how synthetic photometry from Kurucz model spectra can be used to determine the characteristics of ordinary stars. In §2, we describe the SDSS photometric system and the differences between this system and other photometric systems. In §3, we describe the spectra and method for our photometry synthesis, using: the stellar spectrophotometric atlas of Gunn & Stryker (Gunn & Stryker 1983, hereafter GS), white dwarf spectra (Greenstein & Liebert 1990), and stellar model atmosphere spectra (Kurucz 1991). In §4, we compare our synthetic colors with data taken using $u'g'r'i'z'$ filters. We present the results of our photometric metallicity separation analysis for G stars in §5; in §6, we discuss the photometric separation of low-gravity A stars. Section 7 summarizes our work and conclusions.

2. The SDSS Photometric System

F96 provides the detailed definition of the SDSS photometric system; the following is a brief description of the system. The u' , g' , r' , i' , and z' filters have effective wavelengths of 3500 Å, 4800 Å, 6200 Å, 7600 Å, and 9000 Å, respectively (Fig. 1). Magnitudes are AB magnitudes as defined by Oke & Gunn (1983, hereafter OG): the monochromatic AB

magnitude is

$$AB_\nu = -2.5 \log f_\nu - 48.6 , \quad (1)$$

where f_ν has units of $\text{ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}$. The formal definition of the system is such that a flat spectrum (constant f_ν) has $AB = V$. The constant, 48.6, in the above definition provides for conversion from AB magnitude to absolute flux. The formally defined zeropoint of the AB system is such that a flat spectrum has all colors equal to 0. This is a physically meaningful definition because the ratios of constant- f_ν fluxes in different bandpasses are unity; the system definition introduces no artificial zeropoint.

The SDSS filters differ from the “standard” $UBVRI$ filters (which we will take to be those discussed by Bessell [1990]) in several ways. Figure 2 shows the SDSS passbands and Bessell’s $UBVRI$ passbands. While the u and U filters resemble each other somewhat, none of the other “standard” filters are very similar to any of the SDSS filters. The SDSS system covers a wider wavelength range (3000–11500 Å) than the standard system range (3000–9200 Å). Unlike the standard $UBVRI$ passbands, the SDSS passbands overlap relatively little, making the SDSS photometry a more clearly defined diagnostic of spectral features. We refer the reader to F96 for transformations between the $u'g'r'i'z'$ and $UBVRI$ systems.

The $u'g'r'i'z'$ data will be calibrated using low-metallicity F subdwarfs as secondary spectrophotometric standards. These stars are desirable standards because they are reasonably bright for modern large telescopes and have fairly flat spectra with few features (OG). See OG and F96 for a description of the standards and calibration procedure and F96 for discussion of the calibration of the SDSS system.

3. Synthetic Colors

We have calculated synthetic colors on the SDSS system for several sets of real and simulated stellar spectra, as summarized in Table 1:

1. The Gunn-Stryker spectrophotometric atlas (GS) provides spectra covering 3130–10800 Å for stars spanning the H-R diagram. Thirteen of the GS stars are missing data in a wavelength bin of width ≈ 300 Å centered on $\lambda \approx 8000$ Å; we omit these stars from our sample.

2. White dwarf spectra covering 3571–8300 Å are found in Greenstein & Liebert (1990); we extrapolate to cover the $u'g'r'i'z'$ wavelength range using linear fits to the flux values at the shortest three given wavelengths (for the blue end) and to the flux values at the longest three given wavelengths (for the red end). While the use of spectra with complete wavelength coverage would be preferable, we nevertheless consider the resulting approximations to the positions of white dwarfs in color space valuable for estimating the amount of expected overlap of white dwarfs and normal stars in color space. Such overlap must be considered when investigating photometric separations in color space of the type we discuss in §§5 and 6 below.

3. Simulated stellar spectra are provided by the model stellar atmospheres of Kurucz (1991), covering a wide range of effective temperatures, surface gravities, and metallicities. Here, we synthesize colors for atmospheres with T_{eff} and $\log g$ characteristic of main-sequence (dwarf) and giant stars with $[M/H] = 0, +1, -1, -2, \text{ and } -5$, where $[M/H]$ denotes the metallicity relative to solar:

$$[M/H] \equiv \log[(M/H)_{\text{model}}/(M/H)_{\odot}] ; \quad (2)$$

(M/H) denotes metallicity relative to hydrogen. We also synthesize colors for model spectra corresponding to horizontal branch stars and OB subdwarfs.

The synthetic broadband *AB* magnitude is defined as

$$m = -2.5 \log \frac{\int f_\nu S_\nu d(\log \nu)}{\int S_\nu d(\log \nu)} - 48.6 , \quad (3)$$

where f_ν is flux per unit frequency and S_ν is the system response (see eq. [7] of F96). The magnitude is normalized for consistency with the formal definition of the monochromatic magnitude. Note that the argument of the logarithm is proportional to the photon count: since the SDSS filters are designed for use with CCDs rather than with photographic plates, the SDSS system is defined using photon counts rather than energy flux.

The filter responses we use in our calculations are those for the SDSS Monitor Telescope with atmospheric extinction at 1.2 airmasses (Fig. 1), as the Monitor Telescope will be the instrument used to make the system-defining observations (F96).

Tables 2–15 provide the synthetic colors for each of the data sets listed in Table 1. These synthetic colors are to be used with the caveat that the algorithm above is an idealization of the photometric process. Actual observations of standard stars will define the SDSS photometric system.

In Figure 3, we show how the stellar colors vary as a function of temperature, surface gravity, and metallicity in the three $u'g'r'i'z'$ color projections. Figure 3 shows the colors for stars of two surface gravities, $\log g = 4.0$ and $\log g = 2.5$, corresponding roughly to main-sequence stars and giants, respectively; and three metallicities: $[M/H] = +1, 0$, and -5 , to span the high-metallicity to zero-metallicity range. We note that most of the surface gravity and metallicity variation appears in Figure 3a, with some metallicity separation evident in Figure 3b; Figure 3c separates surface gravities of A stars and metallicities of M stars. The bluer colors show the most metallicity separation because most of the line-blanketing from heavy elements occurs in the shorter wavelength regions. The $u'g'r'$ wavelength region contains the Balmer jump, a stellar spectral feature sensitive to surface gravity; hence, the variation due to surface gravity is manifest primarily in the $u' - g'$ vs.

$g' - r'$ projection (Fig. 3a). We note that the metallicity width of the stellar locus in the *UBVRI* system (see, e.g., Mihalas & Binney 1981) is comparable to that in the $u'g'r'i'z'$ system, so the two systems are roughly equivalent metallicity diagnostics in principle, although the expected high level of precision in the SDSS system will be essential for determining metallicities using SDSS photometry. We discuss photometric metallicity separation further in §5.

Figure 4 shows the location of white dwarfs in color space relative to Kurucz models with $\log g = 4.0$ and $[M/H] = +1, 0, \text{ and } -5$. Figure 4a shows the least overlap of the white dwarf and normal star loci; Figures 4b and 4c show considerable overlap. The majority of the white dwarfs can be distinguished from normal stars by their ultraviolet excess, as has been found with other filter systems (Green, Schmidt, and Leibert 1986).

Figure 5 shows the synthetic colors for the GS stars and for Kurucz models with $\log g = 4.0$ and $[M/H] = +1, 0, \text{ and } -5$. GS quote errors in their synthetic *UBVRI* and *uvby* photometry of $\sim 0.02\text{--}0.04$ mag, so most of the scatter in the synthetic photometry of GS stars is likely to be intrinsic. Most of the GS colors are consistent with the Kurucz model colors; there is only one notable outlier in Figure 5a. Some of the M stars extend to cooler temperatures than the Kurucz models currently cover, since atmospheric modeling of very cool stars is difficult (Kurucz 1979, 1991).

Comparison of Figures 3 and 5 suggests that most GS stars have approximately solar metallicities. The cooler GS stars ($T \approx 4000$ K) show some inconsistencies with the Kurucz models. In the $u' - g'$ vs. $g' - r'$ plot, the cool stars separate slightly more by surface gravity than expected for Kurucz models with similar metallicities. Comparison of the synthetic colors of these stars with the Kurucz model colors suggests that the metallicities of these stars are approximately solar. In the $r' - i'$ vs. $i' - z'$ plot, the redder stars ($i' - z' \sim 0.4$) are more consistent with the lower metallicity models; however, the few red stars in this region

with measured metallicities are nearly solar (Cayrel de Strobel et al. 1997). Resolution of such discrepancies requires careful examination of the metallicity measurement errors, the spectral calibration in the GS atlas, and the physics in the model atmospheres.

4. Comparison with Observations

Richards et al. (1997) have obtained $u^*g^*r^*i^*z^*$ photometry for quasars and accompanying field objects (asterisks, rather than primes, are used to denote the data since the SDSS calibration, observing conditions, and instruments will inevitably differ somewhat from those under which these data were taken). The observations were taken with the 0.6-m Monitor Telescope at Apache Point Observatory in Sunspot, New Mexico, which will be used to take the calibration data for the SDSS data taken with the dedicated 2.5-m SDSS telescope. The data were calibrated with observations of the F subdwarfs BD+26°2606 and BD+17°4708, which are two of the fundamental standard stars that define the SDSS system (F96). Thus, the observations and the simulations are calibrated to the same system. We estimate a 10% total error in the $u^*g^*r^*$ and $g^*r^*i^*$ colors and 20% total error in the $r^*i^*z^*$ colors. Note that the errors in Tables 4 and 5 of R97 are the Poisson errors for the program objects; however, the field stars used here are typically fainter than the program objects. The errors quoted above can be understood by considering the faintness of the field stars compared to the program objects and the additional errors described in Table 6 of R97. The larger errors in $r^*i^*z^*$ are probably due to fringing and reduced throughput in the z filter.

Figure 6 shows R97’s observed colors for the (mostly stellar) field objects obtained with the Monitor Telescope together with the synthetic colors for Kurucz model atmospheres with $\log g = 4.0$, $[M/H] = 0$. Figure 6 shows that the synthetic $u'g'r'i'z'$ colors of the model stars are consistent with observed colors. The observed objects are generally of two types:

1) stars from the standard star fields, which were mostly equatorial and had relatively short (10–30 s) exposure times, and 2) stars from quasar fields, which were generally at high galactic latitude and had longer (1–3 minutes) exposure times. The limiting magnitudes for the short-exposure-time fields were $u^*, z^* \sim 16$ and $g^*, r^*, i^* \sim 17.25$, while the limits for the longer exposures are somewhat fainter. Thus, the sample of observed stars is fairly heterogeneous in metallicity, color, and magnitude. The outliers in Figure 6 may be nonstellar objects (such as galaxies), stars with unusual colors, or observational aberrations (such as stars contaminated with cosmic rays). Careful photometric and spectroscopic analysis of such data is required to identify all objects. One of the prerequisites of the SDSS is the determination of the probability that an object with given colors is a given type of object, in order to optimize spectroscopic fiber allocation based on photometry.

In addition to photometric errors, interstellar reddening could be a possible source of errors and discrepancies in Figure 6. The GS spectra used to synthesize the colors are dereddened, but R97’s observed colors are not dereddened. The effect of reddening on $u'g'r'i'z'$ colors must be determined observationally.

5. Photometric Metallicity Separation

5.1. Nonlinear Principal Component Analysis

As we will show, stellar metallicity separation analysis benefits from a more sophisticated analysis than simple examination of the color projections (Fig. 3). We note that the $r' - i'$ vs. $i' - z'$ projection (Fig. 3c) shows little metallicity separation for all but the coolest stars and thus we will not consider it for this purpose.

In order to study the locus in three dimensions, we perform what amounts to a nonlinear principal component analysis on the synthetic colors of the Kurucz models given

in Tables 4–15. We will use the notation of Newberg & Yanny (1997, hereafter NY97). They developed an algorithm which fits loci that are more or less one-dimensional; however, the locus of our synthetic colors looks more like a sheet (i.e., a two-dimensional surface embedded in multidimensional color space) because we have simulated stellar colors for a wide range of metallicities and because the selected grid of model temperatures, surface gravities, and metallicities is not weighted by any expected observational distribution. Therefore, we use a modified version of the NY97 algorithm: we generate the locus fit by selecting a set of $[M/H] = -1.0$ model stellar colors from the approximate center of the distribution. For stars cooler than ~ 5000 K, the $\log g = 1.0$ curve is closest to the locus center; for stars hotter than ~ 7000 K, the $\log g = 3.0$ curve fits best. We use points from the $\log g = 1.5, 2.0,$ and 2.5 models to make a smooth transition from $T = 5000$ K to 7000 K.

Once the locus points are chosen, a set of three mutually perpendicular unit vectors, \hat{k} , \hat{l} , and \hat{m} , is found for each point, with \hat{k} indicating the direction of greatest variation, \hat{l} the direction of greatest variation perpendicular to \hat{k} , and \hat{m} the direction of least variation perpendicular to \hat{k} . Figure 3 shows the locus center, as defined by the adopted locus points, as well as several values of k for reference; k roughly parameterizes the locus “length,” or temperature variation. The \hat{k} unit vector at a given locus point is in the direction from the closest bluer locus point to the closest redder locus point. We assume that the thin direction along the entire locus is approximately $\hat{t} \equiv (0.125, -0.585, 0.801)$ since we know from previous studies that this will produce a reasonable result. This unit vector is not in the same direction as \hat{m} , since \hat{m} is required to be perpendicular to \hat{k} . The unit vector $\hat{l} = \hat{t} \times \hat{k}$ lies along the wider axis of the locus cross section. The unit vector $\hat{m} = \hat{k} \times \hat{l}$ lies along the thinner axis of the locus cross section. To assign values of k , l , and m to a synthetic star, one first determines the locus point in multicolor space closest to the colors for that star; l and m are the distances from this closest point in the \hat{l} and \hat{m} directions,

respectively, and k is the distance in the \hat{k} direction between the model colors and the closest locus point plus the value of k corresponding to that locus point.

Figure 7 shows the variation in the cross section of the locus as a function of the distance along it, parameterized by k . Stars with $1.3 < k < 2.0$ ($T \sim 8000$ K) are primarily separated by surface gravity (Fig. 7b), and stars with $2.5 < k < 3.6$ ($T \sim 5000$ K) are primarily separated by metallicity (Figs. 7d and 7e). In between (Fig. 7c), the width of the distribution is due to a combination of surface gravity and metallicity. These results apply to dereddened stars or stars for which there is very little reddening.

These results partially corroborate the NY97 results for observations of bright stars in *WBVR* filters. In particular, the Kurucz model colors predict that the stellar locus for a set of stars with a variety of temperatures, surface gravities, and metallicities will be a ribbon in $u' - g'$, $g' - r'$, $r' - i'$ multicolor space. The locus is quite thin for stars hotter than ~ 4500 K, while the very red end of the locus, which is populated by M stars, can broaden considerably, depending on the stellar parameters (Fig. 7f). The width of the locus of G and K stars is primarily determined by the metallicities of the individual stars in the sample. We further note that the width of the locus for A stars is primarily determined by surface gravity. We suspect that the tight distribution in Figure 8 of NY97, which shows $[M/H]$ vs. cross-locus distance for a sample of F and G dwarfs, was obtained in part because the dwarfs exhibit a narrow range of surface gravities. NY97 probably could not note this from their sample, since hotter stars were almost exclusively dwarfs and cooler stars were almost exclusively giants; thus, the broadening of the locus of bright *WBVR* stars by surface gravity is substantially reduced.

5.2. Metallicity Separation for G Stars

As Figures 3 and 7 show, most of the cross-locus variation is due to metallicity in the range $R \equiv 0.5 < g' - r' < 0.8$ ($2.5 < k < 3.6$), corresponding roughly to G stars. For $g' - r' < 0.5$ and $g' - r' > 0.8$, the stellar colors are either insensitive to metallicity or the effects of metallicity on the colors cannot be separated from the effects of surface gravity.

We focus on l as the component showing the greatest sensitivity to metallicity. We calculate l in the range R using the formula $l \cong (\vec{r} - \vec{r}_0) \cdot \hat{l}$, where $\vec{r} \equiv (u' - g', g' - r', r' - i')$, $\vec{r}_0 = (1.799, 0.648, 0.238)$ is a point near the center of the locus region corresponding to R , and $\hat{l} = (-0.436, 0.693, 0.574)$ is the direction of \hat{l} at that point. In Figure 8 we show metallicity as a function of l for $[M/H] = +1.0$ to -5 in region R . The width of the distribution may reflect error in the Kurucz models as well as intrinsic spread in the stellar locus; modeling error, however, is likely to induce mostly systematic errors and therefore is unlikely to affect significantly the relative colors (Kurucz 1979). Stars with $l \lesssim -0.05$ have solar or higher metallicity. Stars with very low metallicities occupy the range $l \gtrsim 0.19$. In the range $-0.05 < l < 0.19$, the synthetic colors of the Kurucz models suggest that we can determine the metallicity to ± 0.5 dex for the theoretical stellar “sample” we have selected. The width depends on the distribution of surface gravities and will thus be a function of observational sample. For example, a bright sample might contain only giants at G, and a faint sample might contain essentially all dwarfs; if the sample contained only stars with $4.0 \leq \log g \leq 4.5$, the width of the distribution would be roughly half of that shown in Figure 8.

Cayrel de Strobel et al. (1997) list $[Fe/H]$ values for 37 of the GS stars; of these, nine fall in the range $0.5 < g' - r' < 0.8$, and we show them as diamonds in Figure 8. If more than one metallicity is given, we plot all the reported values. The data are in rough agreement with the models, except for one outlier at $l = -0.245$. Unfortunately, these data

only test the high-metallicity region of the distribution.

We note that there will be some overlap of the distribution in Figure 8 with white dwarfs, unusual stars, and nonstellar objects; the extent of such overlap must eventually be determined observationally. However, Figure 8 can provide valuable guidance for photometric analysis of the large quantities of stellar data that the SDSS will produce. For example, such information might be used to tentatively identify objects with $l > 0.19$ as very low-metallicity stars and tag them for follow-up.

If one were to exclude the $r' - i'$ color from consideration (using $\hat{l} = (-0.533, 0.846, 0.0)$), then the resulting distribution of $[M/H]$ vs. $(\vec{r} - \vec{r}_0) \cdot (\hat{l})$ has more than twice the width of the distribution in Figure 8. This increase in width degrades the accuracy with which metallicities can be assigned; in particular, the high metallicity dwarfs and $[M/H] = -1$ giants overlap in $(\vec{r} - \vec{r}_0) \cdot (\hat{l})$. It is critical for this method of metallicity estimation that photometry be obtained in all four $u'g'r'i'$ filters.

6. Photometric Surface Gravity Separation for A Stars

Inspection of Figures 3 and 7 suggests that stars in the range $-0.15 < g' - r' < 0.00$ (corresponding roughly to A stars) can be well separated by surface gravity using the $u' - g', g' - r', r' - i'$, and $i' - z'$ colors. To find the optimal direction for separating A stars by gravity, we estimate the gradients of $\log g, T$, and $[M/H]$ in this region; we seek multicolor components that maximize the change in $\log g$ while minimizing the changes in T and $[M/H]$.

We use the color of the model star with $\log g = 3$, $[M/H] = -1$, and $T = 7750$ K as a reference point. We estimate the direction of local surface gravity gradient by comparing the colors of that star with the colors of the model with $\log g = 4$, $[M/H] = -1$, and

$T = 7750$. The directions of the temperature and metallicity gradients for A stars are estimated in a similar manner. The gradient unit vectors are:

$$\hat{g} \equiv \frac{\nabla \log g}{\|\nabla \log g\|} = -0.875\hat{x}_{u'-g'} + 0.409\hat{x}_{g'-r'} + 0.182\hat{x}_{r'-i'} - 0.186\hat{x}_{i'-z'} , \quad (4)$$

$$\hat{T} \equiv \frac{\nabla T}{\|\nabla T\|} = -0.107\hat{x}_{u'-g'} - 0.867\hat{x}_{g'-r'} - 0.482\hat{x}_{r'-i'} - 0.075\hat{x}_{i'-z'} , \quad (5)$$

$$\hat{M} \equiv \frac{\nabla[\text{M}/\text{H}]}{\|\nabla[\text{M}/\text{H}]\|} = 0.952\hat{x}_{u'-g'} + 0.117\hat{x}_{g'-r'} - 0.234\hat{x}_{r'-i'} - 0.157\hat{x}_{i'-z'} , \quad (6)$$

where \hat{x}_{y-z} is a unit vector in the direction of variation in the $y-z$ color. We are seeking a unit vector which is perpendicular to \hat{T} and \hat{M} , and as closely aligned with \hat{g} as possible. In order to find this vector, we first solve for the unit vector, \hat{N} , which is normal to \hat{g}, \hat{T} , and \hat{M} :

$$\hat{N} = -0.112\hat{x}_{u'-g'} + 0.348\hat{x}_{g'-r'} - 0.697\hat{x}_{r'-i'} + 0.616\hat{x}_{i'-z'} ; \quad (7)$$

\hat{N} lies in the direction of least variation in $\log g$, T , and $[\text{M}/\text{H}]$.

To find the optimal direction for color separation by surface gravity, we then construct the vector which is perpendicular to \hat{T}, \hat{M} , and \hat{N} :

$$\hat{v} = 0.283\hat{x}_{u'-g'} - 0.354\hat{x}_{g'-r'} + 0.455\hat{x}_{r'-i'} + 0.766\hat{x}_{i'-z'} . \quad (8)$$

Figure 9 shows the separation by gravity along this unit vector. There is a tight correlation for $-0.15 < g' - r' < 0.00$, with a looser correlation if the $g' - r'$ range is extended to $-0.20 < g' - r' < 0.25$. Accurate measurements of other stellar properties allow better constraint of $\log g$; for example, metallicity measurement would result in a tighter limit on the value of $\log g$. This relation, of course, applies to unreddened or dereddened stars only.

Eight of the GS stars have $-0.15 < g' - r' < 0.00$; five of these have spectroscopic luminosity classes listed in Table 2. The values of $r' \cdot \hat{v}$ and luminosity classes for these stars are: 0.219 (?), 0.220 (V), 0.226 (III), 0.232 (?), 0.237 (?), 0.238 (IV), 0.251 (IV), and 0.293 (V). These data suggest that there is not much difference between the surface gravities of A

stars with different luminosity classes. In fact, the outlier with the lowest computed surface gravity ($\log g \sim 2.3$) is classified as a dwarf. From Figure 9, we surmise that the other seven stars have surface gravities of $2.7 < \log g < 3.7$. The failure of photometric colors to distinguish between A giants and dwarfs by surface gravity is consistent with the findings of Newberg and Yanny (1998), who noticed that many AIII stars appear on the main sequence in H-R diagrams of field stars with absolute magnitudes computed using parallaxes from the Hipparcos satellite. The measured range of surface gravities is somewhat lower than expected (e.g., Allen 1973). Resolution of discrepancies among various determinations of $\log g$ requires examination of determinations of $\log g$ via, e.g., luminosity class and stellar modeling.

We note that only 3 of the 37 simulated white dwarfs in $-0.20 < g - r < 0.25$ have $\vec{r} \cdot \hat{v} > 0.1$, suggesting that white dwarfs and normal stars may be separated in this region of multicolor space. The discrimination of white dwarfs in a sample is dependent on good u' band data; white dwarfs show the most separation from the normal star locus in $u' - g'$ (Fig. 3a).

If fewer than 4 colors are available for a given data set, stars can still be separated by surface gravity, especially if $u' - g'$ or $i' - z'$ data are available. To find the direction which separates surface gravities for data with only $g'r'i'z'$ data, for example, we find

$$\hat{v}_{g'r'i'z'} \equiv \frac{\hat{v} + 2.52\hat{N}}{\|\hat{v} + 2.52\hat{N}\|} = 0.193\hat{x}_{g'-r'} - 0.481\hat{x}_{r'-i'} + 0.855\hat{x}_{i'-z'} . \quad (9)$$

The correlation of this component with $\log g$ is also high, but the separation by surface gravity is reduced by almost a factor of three compared with the four-dimensional solution. This follows from the fact that $|\hat{v} \cdot \hat{g}| = 0.45$, while $|\hat{v}_{g'r'i'z'} \cdot \hat{g}| = 0.17$. Similarly, one can construct $\hat{v}_{u'g'r'i'}$, which again produces a good correlation with $\log g$, but with $|\hat{v}_{u'g'r'i'} \cdot \hat{g}| = 0.29$.

7. Conclusions

We present synthetic $u'g'r'i'z'$ photometry for Kurucz model stellar spectra and for white dwarf and Gunn-Stryker spectrophotometry. The synthetic colors of the models show qualitative agreement with the few published observations in these filters. Of the colors we study, $u' - g'$ shows the most differentiation due to stellar properties; the $g' - r'$, $r' - i'$, and $i' - z'$ colors show less dramatic separation. We demonstrate that synthetic $u'g'r'i'z'$ photometry for modeled stars with a range of effective temperatures, surface gravities, and metallicities can provide guidance for photometrically locating stars with certain properties.

The synthetic colors indicate that white dwarfs and normal stars overlap in some color projections. Separation of white dwarfs and normal stars will be most successful using $u' - g'$ data, since white dwarfs separate the most from normal stars in $u' - g'$ vs. $g' - r'$.

The synthetic colors of model atmospheres are consistent with those of Gunn-Stryker stellar spectra; the Gunn-Stryker colors lie mostly within the model stellar locus. The Gunn-Stryker colors include a “tail” of M stars not present in the model colors; since the modeling of such cool atmospheres contains known difficulties (i.e. molecular bands), some discrepancy is not surprising.

Because the stellar locus is basically two-dimensional through most of color space, it is not possible to simultaneously separate stars by temperature, metallicity, and surface gravity. There are, however, special cases in which the locus shows color separation due to variation in a single stellar characteristic. In the range $0.5 < g' - r' < 0.8$, we use a parameterization of the stellar locus to determine the metallicities of stars to within about 0.5 dex. A star in the range $18 \lesssim V \lesssim 20$, near the limit of high accuracy photometry in the SDSS, is either a G–K dwarf at 3–10 kpc, or a G giant at 30–80 kpc. Therefore we may assume that the vast majority of these stars are dwarfs, and use our metallicity relationship to trace the metal abundance as a function of position in the galactic halo. Other

applications for photometric metallicity separation include tagging very-low-metallicity stars for spectroscopic follow-up.

In the range $-0.15 < g' - r' < 0.00$, we use unit vectors approximating the gradients in color space of metallicity, temperature, and surface gravity to develop a relation which best separates A stars by surface gravity. If the synthetic photometry is a reliable simulation of good photometric data, then it is possible to separate unreddened A stars by surface gravity and metallicity (Fig. 7b), though the metallicity separation for A stars is only a few percent in color space even for a large range of metallicities.

We have discovered discrepancies in several cases between synthetic colors of Gunn-Stryker stars, Kurucz model atmospheres, and external measurements of metallicities and surface gravities. These discrepancies could be explained by systematic error of $\sim 5\%$ in the colors of either the Gunn-Stryker stars or the Kurucz models, except at the very red end of the locus where the discrepancy is larger. Careful study of the various observational and theoretical data sets is required to resolve such discrepancy.

One can use techniques similar to those described above, along with the tables of synthetic photometry, to generate relations for other special cases. For example, if one had a sample of F stars that was known by other means to be dwarfs, then the two-dimensional locus could be used to determine the metallicity and temperature of the stars. Such algorithms can facilitate the analysis of large amounts of photometry that will be produced by surveys such as the SDSS.

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Planetarium. Electronic versions of Tables 2–15 may be obtained by contacting D. Lenz at lenz@oddjob.uchicago.edu.

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Fig. 1.— Transmission curves for the Monitor Telescope $u'r'i'z'$ filters (Fukugita et al. 1996). Dotted curves are for 1.2 airmasses; solid curves are without atmosphere.

Fig. 2.— Transmission curves for the Monitor Telescope $u'r'i'z'$ filters without atmosphere (solid curves; Fukugita et al. 1996) and the standard $UBVRI$ filters (dashed curves; Bessell 1990). The $UBVRI$ transmission curves are arbitrarily normalized to a peak transmission of 1.

Fig. 3.— Variation in the position of the stellar locus due to temperature, surface gravity, and metallicity in (a) $u' - g'$ vs. $g' - r'$, (b) $g' - r'$ vs. $r' - i'$, and (c) $r' - i'$ vs. $i' - z'$. Solid curves indicate Kurucz models (Kurucz 1991) with $\log g = 4.0$; dashed curves indicate Kurucz models with $\log g = 2.5$. Three metallicities are shown, resulting in three pairs of solid/dashed curves labeled with a circle for $[M/H] = +1.0$, a box for $[M/H] = 0.0$, and a triangle for $[M/H] = -5.0$. Values of T are marked along the locus; solid lines indicate isotherms for the $\log g = 4.0$ models, and dot-dashed lines indicate isotherms for the $\log g = 2.5$ models. The dotted line is the locus fit; the values of principal component k at the points corresponding to the cross-sectional k ranges shown in Fig. 7 are labeled and marked with large dots.

Fig. 4.— Location of white dwarfs (pluses; Greenstein & Liebert 1990) relative to Kurucz models with $\log g = 4.0$ and $[M/H] = +1.0, 0.0, -5.0$ (solid curves labeled as in Fig. 3; Kurucz 1991). White dwarfs designated DA in the Greenstein & Liebert atlas are circled.

Fig. 5.— Comparison of synthetic colors for Gunn-Stryker stars (diamonds and X's; Gunn & Stryker 1983) with Kurucz model atmospheres for $\log g = 4.0$, $[M/H] = +1.0, 0.0, -5.0$ (solid curves labeled as in Fig. 3; Kurucz 1991). The size of the diamond indicates the luminosity class tabulated by GS.

Fig. 6.— Comparison of observed colors (dots) for field objects of Richards et al. (1997)

with synthetic colors for Kurucz model atmospheres for $\log g = 4.0$, $[M/H] = +1.0, 0.0, -5.0$ (solid curves labeled as in Fig. 3; Kurucz 1991). For simplicity, we denote all colors with superscript primes, although the observed colors are technically denoted with superscript asterisks (see §4).

Fig. 7.— The locus of Kurucz model atmospheres (Kurucz 1991) as a function of the nonlinear principal color components k , l , and m . The panels contain cross sections of the stellar locus from bluest (a) to reddest (f), labeled by k range. The values of k along the locus are given in Figure 3. The symbol shapes indicate metallicity and the symbol sizes indicate surface gravity as shown in the legend in the top panel. Note that the symbols separate roughly by size (surface gravity) in panel b and by shape (metallicity) in panel e.

Fig. 8.— Metallicity separation as a function of $(\vec{r}-\vec{r}_0)\cdot(\hat{l}) \cong l$ for Kurucz model atmospheres (Kurucz 1991) with $0.5 < g' - r' < 0.8$. Symbol sizes and shapes are as in Fig. 7; additional symbols are filled circles: $[M/H] = +0.5$, asterisks: $[M/H] = -0.5$, X's: $[M/H] = -1.5$, filled stars: $[M/H] = -3$, and filled triangles: $[M/H] = -4$. The diamonds are (multiple) measurements of the metallicities of nine Gunn-Stryker stars from Cayrel de Strobel et al. (1997). Smaller diamonds are for a star with $\log g \geq 3.5$.

Fig. 9.— Surface gravity separation as a function of $\vec{r} \cdot \hat{v}$ (see §6 for discussion) for Kurucz model atmospheres (Kurucz 1991) with $-0.15 < g' - r' < 0.00$ (larger circles) and $-0.20 < g' - r' < 0.25$ (smaller circles).

















