Case Study:
Sigma Pie Company

MATP4700/ DSES4770 Math Models of Operations Research
Fall 2008

Students may work in groups of up to three people. You may consult only your textbooks, your notes, and me.

1 Introduction

Sigma Pie Company (SigmaPi) makes many kinds of pies. We are going to focus on their apple pies. In particular, we will look at their name-brand apple pies and their generic pies, and the supply chain involved in getting these pies to customers. The aim is to create an optimal week-by-week supply, production, and distribution schedule for a period of six weeks.

2 Converting apples into pies

We will focus on just two ingredients, namely apples and wheat. SigmaPi obtains apples from three suppliers, one in Washington state, one in Michigan, and one in New Zealand. Each week, the supplier in New Zealand can provide up to 280 cartons at a cost of $200 each, the one in Washington can supply up to 250 cartons for $400 each, and the one in Michigan can supply up to 210 cartons for $500 each. Wheat is obtained from Washington and Michigan, with up to 250 bushels available from Washington and up to 340 bushels available from Michigan each week. The cost is $200 per bushel from each supplier.

SigmaPi has five factories producing Apple Pies. 120 boxes of name-brand pie require 3 cartons of apples and 2 bushels of wheat. 100 boxes of generic apple pie require 1 carton of apples and 1 bushel of wheat. You can assume that fractional cartons can be used, and fractional boxes can be produced. The factories, their weekly capacities in raw apples, and their initial inventories are as given in Table 1. It takes one week for a crate of apples to get from New Zealand to any of the factories, at a cost of $300. Crates and bushels can be shipped from Washington to the factories in St Paul, Seattle, or Sacramento. Crates and bushels can be shipped from Michigan to the factories in St Paul, Chattanooga, or Buffalo. You can assume that getting a carton of apples or a bushel of apples from either Washington or Michigan to any of the factories can be done effectively instantly, at a cost of $100.

	Capacity	Apple Inventory	Wheat Inventory
St Paul	260	95	70
Seattle	200	60	45
Sacramento	240	90	50
Chattanooga	240	90	60
Buffalo	200	65	45

Table 1: Weekly capacities and initial inventory of the factories, in cartons of apples. Note that the initial inventory includes apples shipped from New Zealand in the previous week, but not apples shipped from Washington or Michigan in the first week.

	1 NB	2 NB	3 NB	4 NB	5 NB	6 NB	1 G	2 G	3 G	4 G	5 G	6 G
Dallas	20	10	10	15	15	10	50	40	60	55	45	50
Salt Lake City	30	30	30	35	35	30	30	30	30	35	35	30
Toronto	20	20	20	25	25	20	20	20	20	25	25	20
Kansas City	35	45	25	30	30	35	55	45	65	50	40	55
Fresno	10	20	20	15	25	10	10	20	20	15	25	10
Birmingham	40	40	20	15	25	40	40	40	20	45	25	40
Philadelphia	20	30	20	25	35	30	20	30	40	35	35	30

Table 2: Weekly demand for boxes of name-brand(NB) and generic (G) apple pies, in hundreds

3 Shipping and selling the pies

The pies are shipped to seven distribution centers, where the boxes of name-brand pies are sold for $37 each, and the boxes of generic pies are sold for $16 each. The distribution centers and their estimated weekly demands are contained in Table 2. If SigmaPi is unable to meet this estimated demand, they incur a cost of $5 for each name-brand box and $3 for each generic box, corresponding to the cost of maintaining goodwill with customers by offering rainchecks and using other techniques.

It can be assumed that if a box of pies is produced in week t then it is available at the distribution center in week $t+1$. There is a shipping cost associated with shipping the pies. The cost differs between the name-brand and generic pies, because the name-brand pies are larger and also because the generic pies contain more preservatives and so don't have to be kept quite so cold. Each distribution center is only served by some of the factories. The shipping costs are in Tables 3 and 4.

	St Paul	Seattle	Sacramento	Chattanooga	Buffalo
Dallas	7	-	7	8	-
Salt Lake City	6	8	7	-	-
Toronto	5	-	-	-	7
Kansas City	6	7	7	7	-
Fresno	-	7	6	-	-
Birmingham	8	-	-	6	6
Philadelphia	-	-	-	7	7

Table 3: Shipping costs per box of name-brand apple pies

	St Paul	Seattle	Sacramento	Chattanooga	Buffalo
Dallas	4	-	4	4	-
Salt Lake City	3	4	4	-	-
Toronto	3	-	-	-	4
Kansas City	3	4	4	4	-
Fresno	-	4	3	-	-
Birmingham	5	-	-	3	3
Philadelphia	-	-	-	4	4

Table 4: Shipping costs per box of generic apple pies

	Name Brand	Generic
Dallas	2000	5000
Salt Lake City	3000	3000
Toronto	2000	2000
Kansas City	3500	5500
Fresno	1000	1000
Birmingham	4000	4000
Philadelphia	2000	2000

Table 5: Initial inventories of boxes of pies at the distribution centers

4 Inventory issues

At the beginning of each week, a factory receives a shipment of apples and wheat. It uses these raw materials as well as raw materials already on hand to produce pies over the course of the week, and to ship these pies to the distribution centers at the end of the week. Some of the pies can be kept at the factory and shipped out later.

There is some uncertainty in the demands. Therefore, SigmaPi chooses to keep a safety stock of apples and wheat at each factory. They have determined that appropriate levels are 20 cartons and bushels at St Paul and Sacramento, 15 cartons and bushels at Seattle and Buffalo, and 10 cartons and bushels at Chattanooga.

There is a cost of $10 for each carton of apples or bushel of wheat kept in inventory at a factory. There are also costs of $0.20 and $0.10 for each box of name-brand and generic pies, respectively, kept in inventory, either at a distribution center or at a factory.

5 Preventing finite horizon effects

As described so far, an optimal solution would use up all the inventory at the end of the six weeks, and would ship no apples from New Zealand in week 6. This would leave SigmaPi in a difficult situation in subsequent weeks. To avoid this consequence of using a finite horizon, SigmaPi requires that the inventory situation at the end of six weeks should be the same as at the beginning of the period. The initial inventories of apples and wheat at the factories are contained in Table 1. Note that these initial inventories include apples shipped from New Zealand in the previous week. There is no initial inventory of boxes of pies at the factories. Initial inventories of pies at the distribution centers are detailed in Table 5.

6 Questions

1. Due: **Friday, September 26**, in class. (40 points)
   (Please submit a written report, a printout of your problem files, and any AMPL output used. Also email me your model and data files.)

   (a) Formulate a linear program which can be used to generate an optimal shipping and production plan for SigmaPi. Clearly define every variable in your formulation. Explain every constraint including coefficients.

   (b) Solve your linear program using AMPL. Give the amount of apples and wheat purchased, the amount of apples, wheat and pies shipped on each route, and the amount of pies sold. As simply as you can, describe the solution in words. Note: As a check, your profit should be somewhere between $\$10^6$ and $\$2 \times 10^6$.

2. **Sensitivity analysis** (40 points): To be distributed later. Solutions due on October 31.

3. **Extra conditions** (20 points): To be distributed later. Your answers will be due on December 5.

7 AMPL notes

- If you are working in a Windows operating system, you can edit your model and data files in Office, for example. Save the files as plain text files. Windows may append the suffix `txt` to the file names, in which case you would need to include that suffix when asking ampl to read the file.

- If you want to read in a new model file `chips.mod` and data file `chips.dat`, you can use the `reset` command:

```
ampl: reset;
ampl: model chips.mod;
ampl: data chips.dat;
```

- If you want to reset the whole data file and read in a new data file `chips.dat`, but you want to keep the model file as before, you can type

```
ampl: reset data;
ampl: data chips.dat;
```

- If you want to change one parameter, you can use the **let** command as follows:

```
(OS) ampl
ampl: model sample/steel.mod;
ampl: data sample/steel.dat;
ampl: solve;
MINOS 5.4: optimal solution found.
2 iterations, objective 192000
ampl: let rate["bands"]:=250;
ampl: solve;
MINOS 5.4: optimal solution found.
1 iterations, objective 217200
```

This changes the rate for bands and then resolves the problem.

- More hints are available online from my page about AMPL.
- This project is available online. You may find it helpful to cut-and-paste some of the data from the web version to your model.