Precalculus Review Sheet

Completing the Square

$x^2 + 3x + 1 = 0$

Move the integer value to the right: $x^2 + 3x = -1$

Take ½ of the coefficient of x and square it: $(3/2)^2 = 9/4$

Add 9/4 to both sides of the equation: $x^2 + 3x + 9/4 = -1 + 9/4$

You get: $(x + 3/2)^2 = -4/4 + 9/4 = 5/4$ OR $(x + 3/2)^2 - 5/4 = 0$

Synthetic Division

$(3z^4 + 8z^3 + 4z^2 - z - 2)$ divided by $(z + 2)$

		z^4	z^3	z^2	z^1	z^0
		3	8	4	-1	-2
-2	+		-6	-4	0	2
	*	3	2	0	-1	0
		z^3	z^2	z^1	z^0	

The Answer: $(3z^3 + 2z^2 - 1)$

Check: $(3z^3 + 2z^2 - 1)(z + 2) = 3z^4 + 8z^3 + 4z^2 - z - 2$

Long Division

$$
\begin{array}{r}
3z^3 + 2z^2 + 0z - 1 + 0 \; * \\
z + 2 \;\sqrt{3z^4 + 8z^3 + 4z^2 - z - 2} \\
\underline{-3z^4 - 6z^3} \\
0 + 2z^3 + 4z^2 \\
\underline{-2z^3 - 4z^2} \\
0 \;\; + 0 - z - 2 \\
\underline{+z + 2} \\
0 + 0
\end{array}
$$

Sets and Subsets

A = {a, b, c, ...x, y z}

B = {0, 1, 2, 3}

Set A and B are collections of elements.

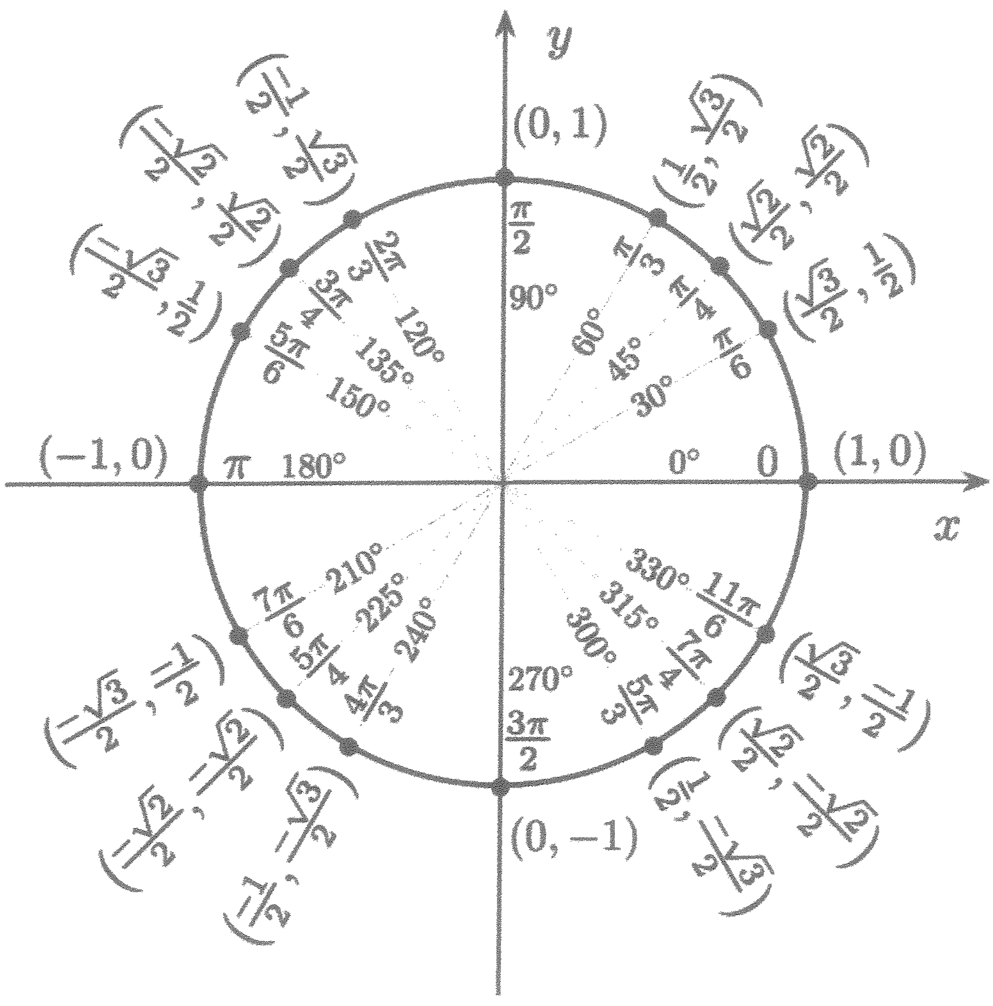
Set B can also be described as the closed interval

$B = \{x \mid 0 \leq x \leq 3\}$

A subset of B is $B_1 = \{x \mid 0 < x < 3\}$

<u>Converting degrees to radians and vice-versa</u>

$$\text{radians} = \text{degrees} \times \frac{\pi}{180} \qquad \text{degrees} = \text{radians} \times \frac{180}{\pi}$$



Quadrant I: 0 to 90 degrees = 0 to $\pi/2$ radians
Quadrant II: 90 to 180 degrees = $\pi/2$ to π radians
Quadrant III: 180 to 270 degrees = π to $3\pi/2$ radians

Quadrant IV: 270 to 360 degrees = $3\pi/2$ to 2π radians
where $\pi = 3.14$ rounded to 2 decimal places.

Quadrant II sin + cos - tan -	Quadrant I sin + cos + tan +
Quadrant III sin - cos - tan +	Quadrant IV sin - cos + tan -

Trigonometric Identities

Defining relations for tangent, cotangent, secant, and cosecant in terms of sine and cosine.

$$\tan t = \frac{\sin t}{\cos t} \qquad \cot t = \frac{1}{\tan t} = \frac{\cos t}{\sin t}$$

$$\sec t = \frac{1}{\cos t} \qquad \csc t = \frac{1}{\sin t}$$

The Pythagorean formula for sines and cosines

$$\sin^2 t + \cos^2 t = 1$$

Other identities

$$\tan^2 \mathrm{t} + 1 = \sec^2 \mathrm{t}$$

$$1 + \cot^2 = \csc^2 \mathrm{t}$$

Identities expressing trig functions in terms of their complements

$$\cos t = \sin(\pi/2 - t) \qquad \sin t = \cos(\pi/2 - t)$$
$$\cot t = \tan(\pi/2 - t) \qquad \tan t = \cot(\pi/2 - t)$$
$$\csc t = \sec(\pi/2 - t) \qquad \sec t = \csc(\pi/2 - t)$$

Periodicity of trig functions. Sine, cosine, secant, and cosecant have period 2π while tangent and cotangent have period π.

$\sin(t + 2\pi) = \sin t$
$\cos(t + 2\pi) = \cos t$
$\tan(t + \pi) = \tan t$

Identities for negative angles. Sine, tangent, cotangent, and cosecant are odd functions while cosine and secant are even functions.

$\sin(-t) = -\sin t$
$\cos(-t) = \cos t$
$\tan(-t) = -\tan t$

Sum formulas for sine and cosine

$\sin(s + t) = \sin s \cos t + \cos s \sin t$
$\cos(s + t) = \cos s \cos t - \sin s \sin t$
$\sin(s - t) = \sin s \cos t - \cos s \sin t$
$\cos(s - t) = \cos s \cos t + \sin s \sin t$

Tangent identities

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double-Angle Formula

$\sin(2x) = 2 \sin x \cos x$
$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

Half-Angle Formula

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

Product Formula

$\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$
$\cos x \cos y = \frac{1}{2}[\cos(x + y) + \cos(x - y)]$
$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$

Properties of Exponents and Logarithms

Logarithms

1. $\log_b x = n$ implies $b^n = x$.

2. $\log_b xy = \log_b x + \log_b y$

3. $\log_b x/y = \log_b x - \log_b y$

4. $\log_b x^n = n \log_b x$

Properties of the exponential function
$e^{x+c} = e^x e^c$
$e^{bx} = (e^b)^x = (e^x)^c$
$e^0 = 1$
$e^{-bx} = \frac{1}{e^{bx}}$
$\ln(e^x) = x$ $e^{\ln(x)} = x$
$a^x = e^{\ln(a)x}$

Natural Logarithm
$\ln(ax) = \ln(a) + \ln(x)$
$\ln(a^x) = x \ln a$
$\ln(1) = 0$
$\ln(1/x) = -\ln(x)$

EXAMPLES

Ex 1 Write in exponential form: $\log_2 32 = 5$

Answer. $2^5 = 32$

Ex 2 Write in logarithmic form: $4^{-2} = 1/16$

Answer. $\log_4 (1/16) = -2$