Corrections to Chapter 8 on Shaft Analysis in Mechanical Design of Machine Elements and Machines by Jack Collins

Equations (8-6), (8-7), and (8-8) are wrong.

In shafts, there are 2 primary sources of stress, bending and torsion. Stresses due to axial loads are also frequently present, but typically negligible. So, assuming just bending and torsion, let M_a and M_m be the alternating and mean bending moments, and let T_a and T_m be the alternating and mean torques, respectively. Frequently, for a steady, rotating shaft (i.e. rotating at constant speed with constant loads) the torque can be treated as steady (i.e. $T_a = 0$) and the moment can be treated as completely reversed due to the shaft rotation (i.e. $M_m = 0$). However, keeping the equations general, we can express the alternating and mean bending stresses (which act along the axis of the shaft) as

$$\sigma_{ba} = \frac{M_a c}{I} = \frac{M_a \frac{d}{2}}{\frac{\pi}{64} d^4} = \frac{32 M_a}{\pi d^3} \quad and \quad \sigma_{bm} = \frac{32 M_m}{\pi d^3} \tag{1}$$

Likewise, for the alternating and mean torsional shear stresses as

$$\tau_a = \frac{T_a c}{J} = \frac{T_a \frac{d}{2}}{\frac{\pi}{32} d^4} = \frac{16 T_a}{\pi d^3} \quad and \quad \tau_m = \frac{16 T_m}{\pi d^3} \tag{2}$$

Now, recall from Table 4.7 that for highly local stress concentrations, for static loadings we don't apply a stress concentration ($K_t \to 1$), and for cyclic loadings, we apply K_f, the fatigue stress concentration factor computed using Eqn. (4-111) (note, q is different for bending and torsion as found in Fig. 4.27, and K_t is determined from the appropriate graph, Figs. 4.17 to 4.25). Thus, no stress concentration factor is applied to the mean part of the stress, and the appropriate fatigue stress concentration factor is applied to each of the alternating stresses. Substituting Eqns. (1) and (2) into the equations for the effective alternating and mean stresss equation (see previous handout on corrections to multiaxial fatigue equations) and applying the appropriate stress concentration factors then results in

$$\sigma_{eq-a} = \frac{16}{\pi d^3} \left[\left(2 K_{fb} M_a\right)^2 + 3 \left(K_{ft} T_a\right)^2 \right]^{\frac{1}{2}} \tag{3}$$

and

$$\sigma_{eq-m} = \frac{16}{\pi d^3} \left[\left(2 M_m\right)^2 + 3 {T_m}^2 \right]^{\frac{1}{2}} \tag{4}$$

where K_{fb} and K_{ft} are the fatigue stress concentration factors in bending and torsion, respectively. Plugging into Eqn. (4-99) in your book, the equivalent completely reversed uniaxial stress is then,

$$\sigma_{eq-CR} = \frac{\sigma_{eq-a}}{1 - \frac{\sigma_{eq-m}}{S_u}} = \frac{S_u \sigma_{eq-a}}{S_u - \sigma_{eq-m}} = \frac{S_u \left[\left(2 K_{fb} M_a\right)^2 + 3 \left(K_{ft} T_a\right)^2 \right]^{\frac{1}{2}}}{\frac{\pi d^3}{16} S_u - \left[\left(2 M_m\right)^2 + 3 {T_m}^2 \right]^{\frac{1}{2}}} \tag{5}$$

The safety factor guarding against fatigue failure is then

$$n_f = \frac{S_N}{\sigma_{eq-CR}} \tag{6}$$

where S_N is the fatigue strength and is equal to S_f for infinite life. Note, since the stress concentration factors were used to adjust the stress in this case, the fatigue strength should not be adjusted by the stress concentration factors (always only adjust one or the other, not both); thus, the fatigue strength modifying factor should be taken to be $k_f = 1$ in this case. For shaft design, Eqns. (5) and (6) can be solved for d giving

$$d = \left(\frac{16}{\pi S_u} \left\{ \frac{n_f S_u}{S_N} \left[(2K_{fb}M_a)^2 + 3\left(K_{ft}T_a\right)^2 \right]^{\frac{1}{2}} + \left[(2M_m)^2 + 3{T_m}^2 \right]^{\frac{1}{2}} \right\} \right)^{\frac{1}{3}} \quad . \tag{7}$$

It is also important to check for yield. Then use the maximum bending moment M_{max} and the maximum torque T_{max} to get the equivalent uniaxial maximum stress

$$\sigma_{eq-max} = \frac{16}{\pi d^3} \left[(2M_{max})^2 + 3{T_{max}}^2 \right]^{\frac{1}{2}} \tag{8}$$

and compute the safety factor against yielding as

$$n_y = \frac{S_{yp}}{\sigma_{eq-max}} \quad . \tag{9}$$