Corrections to Section on Multi-axial Fatigue Analysis in Mechanical Design of Machine Elements and Machines by Jack Collins

Regarding Multi-Axial Fatigue, Chapter 4

The ideas expressed in Section 4.7 are basically correct, however, the application in Example 4.11 is not correct. Also, as mentioned in class, the equivalent uniaxial stress can be expressed in a general form in terms of the components of the stress tensor which eliminates the need for finding the principal stresses. Specifically,

$$\sigma_{eq} = \frac{1}{\sqrt{2}} \left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2 + 6\left(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2\right)\right]^{\frac{1}{2}}$$

is equivalent to Equation (4-96) in the text and does not require solving for the principal stresses first.

When considering a multi-axial fatigue loading (when we say multi-axial, we mean anything beyond uniaxial, i.e. with more than 1 non-zero stress component, for example combined bending and torsion gives an axial stress and shear stress), first, determine what the mean and alternating stresses are on a component-by-component basis. Then, plug the alternating and mean stress components into equations similar to the one above, i.e.

$$\sigma_{eq-a} = \frac{1}{\sqrt{2}} \left[(\sigma_{x-a} - \sigma_{y-a})^2 + (\sigma_{x-a} - \sigma_{z-a})^2 + (\sigma_{y-a} - \sigma_{z-a})^2 + 6\left(\tau_{xy-a}^2 + \tau_{xz-a}^2 + \tau_{yz-a}^2\right)\right]^{\frac{1}{2}}$$

and

$$\sigma_{eq-m} = \frac{1}{\sqrt{2}} \left[(\sigma_{x-m} - \sigma_{y-m})^2 + (\sigma_{x-m} - \sigma_{z-m})^2 + (\sigma_{y-m} - \sigma_{z-m})^2 + 6\left(\tau_{xy-m}^2 + \tau_{xz-m}^2 + \tau_{yz-m}^2\right)\right]^{\frac{1}{2}}$$

where subscripts a and m indicate alternating and mean, respectively. The results from these equations can be directly put into equation (4-99) which can then be compared against the fatigue strength. To check for yielding, just plug in the maximum stress components into Eqn. (1) and check against S_{yp}.

So what is wrong with Example 4.11 and why does it give a different result than the procedure described above, you may ask yourself? The problem is as follows. The author correctly writes the 3 principal stresses in terms of the moment and torque in Eqns. (10), (11), and (12) on page 210, however, for different values of M, which is what is fluctuating in this example, the principal stress axes will be different. Remember, the principal stresses are the normal stresses in the coordinate system where the shear stress is zero. This coordinate system is different for different stress states (i.e. different values of M). You cannot then add and subtract components of the stress tensor (in this case principal stresses) computed with respect to different coordinate systems, which is what the author does in Eqns. (13) - (24).